Metastable vacua in field theory & cosmology

1. Introduction & outline

Applications:

- Tunneling rates in quantum mechanics
- Decay rates of metastable vacua in QFT
  ↳ Lifetime of SM & BSM theories
- First order phase transitions at finite temperature
  ⟶ Stochastic background of gravitational waves

Common approaches & their shortcomings:

- Order-of-magnitude WKB
- Euclidean solitons & fluctuations
- Inclusion of loop corrections is often attempted through use of effective potentials / RG improvement
  ↳ Only strictly valid for homogeneous field configurations at potential minima — not for solitons

  cf. one-loop effective potential

$$V(\phi) = V_0(\phi) + \frac{1}{64\pi^2}\,\mathrm{str}\, m^4(\phi) \log\frac{m^2(\phi)}{\mu^2}$$

  ⟶ only well-defined if $m^2(\phi) > 0$

- One-loop determinants using Gel'fand Yaglom theorem
  ↳ Not clear how to
  - go beyond one loop
  - deal with radiatively generated potentials
  - deal with (approximately) classically scale invariant models (SM?)

- Green's functions for the Euclidean path integral
  ↳ Systematic straightforward loop expansion — see how practicable this is from concrete examples

Goal is to prepare for (resummed) perturbation theory around solitons/instantons. This can be used in phenomenological calculations and in some conceptual developments. While Coleman & Callan have shown the way how to do this in principle, concrete implementation are rare, and we discuss these in the lectures. This is all worth while because God is in the detail. Functional determinants in backgrounds that are not Poincaré invariant are a fascinating piece of theoretical physics. Mastering these, one is prepared to go beyond order-of-magnitude estimates and compute tunneling rates in practice.
These lectures are self contained, i.e. we go through the basics of tunneling theory along with our developments.

Outline:

2. Tunneling rate from the WKB approximation to the Schrödinger equation
3. Tunneling from Euclidean path integrals
   - Euclidean solitons • Zero modes & collective coordinates • negative modes • multi-bounces • decay rate
4. One loop determinants
   Gel'fand-Yaglom method • Resolvent & Green's functions
5. Tunneling in QFT
6. Archetypical QFT example: quasi-degenerate quartic potential