Phase transitions in the Early Universe

2b. Hydrodynamics with a scalar field

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20. syyskuuta 2018

Outline

Fluid energy-momentum tensor

Distribution function for system in local equilibrium:

$$f^{\text{eq}}(p,x) = \frac{1}{e^{\beta(U_\mu p^\mu - \mu)} \pm 1}$$

Energy-momentum tensor:

$$\begin{aligned} T^{\mu\nu} &= \int \frac{\bar{d}^3 p}{2E} 2p^\mu p^\nu f^{\text{eq}}(p,x) \\ T^{\mu\nu} &= (e+p)U^\mu U^\nu + p g^{\mu\nu} \end{aligned}$$

where

$$\begin{aligned} e &= \int \bar{d}^3 p \, E \, f_0^{\text{eq}}(p,x) \quad \text{rest frame energy density} \\ p &= \int \bar{d}^3 p \, \frac{\mathbf{p}^2}{3E} f_0^{\text{eq}}(p,x) \quad \text{rest frame (kinetic) pressure} \end{aligned}$$

EM (non)-conservation for particles with field-dependent mass

$$\left(p^\mu \partial_\mu + mF^\mu \frac{\partial}{\partial p^\mu}\right) \theta(p^0)\delta(p^2 + m^2) f(p,x) = C[f]$$

- $\times$ both sides by p^ν and integrate over momenta
- Assume collisions occur "at a point" and still conserve momentum

$$\tfrac{1}{2}\partial_\mu T^{\mu\nu} + mF^\mu \int \bar{d}^4 p\, p^\nu \tfrac{\partial}{\partial p^\mu} \theta(p^0)\delta(p^2 + m^2) f(p,x) = 0$$

Integration by parts, $F^\mu = -\partial^\mu m = \partial^\mu \bar{\phi}\, dm/d\bar{\phi}$

$$\partial_\mu T^{\mu\nu} = -\partial^\nu \bar{\phi} \frac{dm^2}{d\bar{\phi}} \int \frac{\bar{d}^3 p}{2E} f(p,x)$$

Fluid coupled to scalar field through mass 1

Model for the system near phase transition(1)

$$\begin{array}{lcll} \text{fluid} & T_{\rm f}^{\mu\nu} & = & (e+p)U^\mu U^\nu + p g^{\mu\nu} \\ \text{field} & T_\phi^{\mu\nu} & = & \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 + V_0(\phi)\right) \end{array}$$

- Note: $p = g_{\rm eff}\pi^2 T^4/90 - \Delta V_T(\phi)$ i.e. minus free energy of the fluid
- Conservation of energy-momentum: $\partial_\mu\left(T_{\rm f}^{\mu\nu} + T_\phi^{\mu\nu}\right) = 0$

Hence non-conservation of $T_{\rm f}^{\mu\nu}$ must appear in $T_\phi^{\mu\nu}$

$$\partial_\mu T_\phi^{\mu\nu} \quad = \quad +\partial^\nu\bar\phi\,\frac{dm^2}{d\bar\phi}\int\frac{\bar{d}^3p}{2E}f(p,x)$$

Implies for scalar field equation(2)

$$\Box\phi - V_0'(\phi) = \frac{dm}{d\bar\phi}\int\frac{\bar{d}^3p}{2E}f(p,x)$$

(1) Ignatius, Kajantie, Kurki-Suonio, Rummukainen 1991

(2) Also derivable from field theory, see Moore & Prokopec 1996

Fluid coupled to scalar field through mass 2

$$\Box\phi - V_0'(\phi) = \frac{dm}{d\bar{\phi}} \int \frac{\bar{d}^3 p}{2E} f(p,x)$$

Write $f = f^{\text{eq}} + \Delta f$

$$\Box\phi - V_0'(\phi) = \Delta V_T(\phi) + \frac{dm}{d\bar{\phi}} \int \frac{\bar{d}^3 p}{2E} \Delta f(p,x)$$

Put equilibrium part on LHS:

$$\boxed{\Box\phi - V_T'(\phi) = \frac{dm^2}{d\bar{\phi}} \int \frac{\bar{d}^3 p}{2E} \Delta f(p,x)}$$

Repackage all effective potential into fluid EM: $p \to p = g_{\text{eff}}\pi^2 T^4/90 - V_T(\phi)$

$$\boxed{\partial_\mu T_{\text{f}}^{\mu\nu} + \partial^\nu\phi \frac{\partial V_T(\phi)}{\partial\phi} = -\partial^\nu\phi \frac{dm^2}{d\bar{\phi}} \int \frac{\bar{d}^3 p}{2E} \Delta f(p,x)}$$

IKKR model and entropy generation

$$\Box\phi - V_T'(\phi) = \frac{dm^2}{d\phi}\int \frac{\bar{d}^3 p}{2E}\Delta f(p,x)$$

- Near equilibrium RHS a function of dynamical variables β, U^μ, (μ), ϕ
- Field gradients disturb eqm: expect RHS $\sim \partial_\mu \phi$
- Isotropy: expect RHS $\sim U \cdot \partial\phi$
- Field comes from $m^2(\phi)$ in Δf, so $U \cdot \partial\phi$ should come from $U \cdot \partial m^2$
- Remaining dimensions can be absorbed with T

Suggests ($m^2 \propto \phi^2$):

$$\boxed{\Box\phi - V_T'(\phi) = \eta_T(\phi) U \cdot \partial\phi \qquad \text{with} \qquad \eta_T(\phi) = \tilde{\eta}\beta\phi^2}$$

Can show that entropy generation is always positive Exercise!:

$$\partial_\mu S^\mu = \eta_T(\phi)(U \cdot \partial\phi)^2 \geq 0$$

Entropy current $S^\mu = sU^\mu$, $s = dp/dT$

Summary

- Electroweak symmetry is broken at $T \simeq 100$ GeV
- Standard Model plasma at $T \simeq 100$ GeV:
  weakly-interacting and long-lived W, Z, t, h + “bath" of light particles
- In semi-classical picture SM phase transition is 1st order (just)
- Interactions (non-Abelian gauge bosons) $\to$ cross-over
- Beyond the Standard Model: more scalars $\to$ 1st order phase transition
- Model of coupled order-parameter ϕ and fluid $T_{\rm f}^{\mu\nu}$

$$\Box\phi - V_T'(\phi) = \frac{dm^2}{d\bar{\phi}}\int \frac{\bar{d}^3 p}{2E}\Delta f(p,x) \qquad \simeq \eta_T(\phi)(U\cdot\partial\phi)$$

$$\partial_\mu T_{\rm f}^{\mu\nu} + \partial^\nu\phi\frac{\partial V_T(\phi)}{\partial\phi} = -\partial^\nu\phi\frac{dm^2}{d\bar{\phi}}\int \frac{\bar{d}^3 p}{2E}\Delta f(p,x) \simeq \eta_T(\phi)(U\cdot\partial\phi)\partial^\nu\phi$$

Where $p = g_{\rm eff}\pi^2 T^4/90 - V_T(\phi)$, $\Delta f(p,x) = f(p,x) - f^{\rm eq}(p,x)$

Reading

Statistical physics

- *Statistical Mechanics*, K Huang (Wiley, 1987)

Thermal quantum field theory

- *Basics of Thermal Field Theory*, M. Laine and A. Vuorinen (Springer, 2016) [arXiv:1701.01554]

Relativistic hydrodynamics

- *Relativistic Hydrodynamics*, L. Rezzolla and O. Zanotti (OUP, 2013)

Scalar field coupled to a fluid

- *From Boltzmann equations to steady wall velocities*, T. Konstantin, G. Nardini, I. Rues [arXiv:1407.3132]
- *Energy Budget of Cosmological First-order Phase Transitions*, J.R. Espinosa, T. Konstantin, J.M. No, G. Servant [arXiv:1004.4187]
- *Growth of bubbles in cosmological phase transitions*, J. Ignatius, K. Kajantie, H. Kurki-Suonio, M. Laine [arXiv:astro-ph/9309059]