Phase transitions in the Early Universe

2a. Hydrodynamics in the early Universe

Mark Hindmarsh[1,2]

[1] Department of Physics & Astronomy
University of Sussex

[2] Department of Physics and Helsinki Institute of Physics
Helsinki University

Heraeus School
19. syyskuuta 2018

Outline

Recap: effective potential for first order phase transition

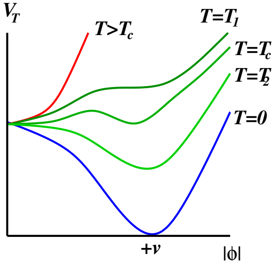
Relativistic hydrodynamics

First order phase transition

Effective potential $V_T(\bar{\phi}) = V_0 + \Delta V_T$

$$\Delta V_T \simeq \frac{D}{2}(T^2 - T_2^2)|\bar{\phi}|^2 - \frac{A}{3}T|\bar{\phi}|^3 + \frac{\lambda}{4!}|\bar{\phi}|^4$$

- Second minimum develops at T_1
- Critical temperature T_c:
  free energies are equal.
- System can supercool below T_c.
- First order transition
  discontinuity in free energy



1st order phase transitions in SM extensions

- 2HDM (2 Higgs doublet model)
    - Extra scalars (A^0, H^0, $H^\pm$) increase strength of cubic term.
    - Strong phase transition when $m_{A^0} \gtrsim 400$ GeV(1)
- Extra singlet scalars
    - Tree level first order phase transition
    - Strong phase transition with SM-like phenomenology allowed(2)
- Effective field theory with h^6 operator(3)
    - e.g. by integrating out singlet(4)
    - $V_T(\phi) \simeq c_0 + c_1(T)h^2 + c_2 h^4 + c_3 h^6 + \cdots$
    - $c_2 < 0$ gives 1st order transition at tree level.
- etc. etc. etc.

---

(1) Dorsch, Huber, No (2015)

(2) Ashoorioon, Konstandin (2009)

(3) Grojean, Servant, Wells (2005)

(4) Huber et al (2006)

Standard Model plasma: semiclassical approximation

	h	$W^\pm$	Z	t
M/GeV	125	80.4	91.2	174
Γ/GeV	4×10^{-3} (*)	2.1	2.5	1.4
d.o.f.	1	6	3	$\frac{7}{8}12$

(*) calculated from SM, not yet measured

- W, Z, t, h have largest mass change: $g_{\text{eff}} = 20.5$
- Each have frequent scatterings with “light” particles $g_{\text{eff}} = 86.25$
- Relatively narrow width of important particles
- Scattering more rapid than decays: semi-classical particles

Relativistic Boltzmann equation

- Distribution function[(5)] (Lorentz scalar): $f(p, x)$
- Average number of particles in phase space volume element at $(\mathbf{p}, \mathbf{x}, t)$
- $p^0 = E_\mathbf{p} = \sqrt{}(\mathbf{p}^2 + m^2)$ is not independent

number density	$n(x)$	$\int \bar{d}^3 p f(p,x)$
particle flux	$j^i(x)$	$\int \bar{d}^3 p \frac{p^i}{E} f(p,x)$
energy density	$e(x)$	$\int \bar{d}^3 p E f(p,x)$
momentum density	$\Pi^i(x)$	$\int \bar{d}^3 p\, p^i f(p,x)$
momentum flux (j direction)	$\Pi^{ij}(x)$	$\int \bar{d}^3 p\, p^i \frac{p^j}{E} f(p,x)$

Organise into 4-vector and 4-tensor:

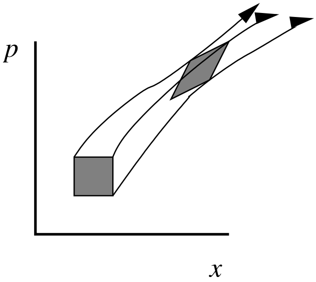
$$j^\mu = \int \frac{\bar{d}^3 p}{2E} 2p^\mu f(p,x) \qquad T^{\mu\nu} = \int \frac{\bar{d}^3 p}{2E} 2p^\mu p^\nu f(p,x)$$

Manifestly covariant form: $\int \frac{\bar{d}^3 p}{2E} = \int \bar{d}^4 p \theta(p^0)\, \bar{\delta}(p^2 + m^2)$

---

(5) Bad notation: not to be confused with free energy density

Particle flow in phase space with forces



$$\begin{aligned} x^\mu &\rightarrow x^\mu + \frac{dX^\mu}{d\tau}\Delta\tau \\ p^\mu &\rightarrow p^\mu + F^\mu \Delta\tau \end{aligned}$$

Force must preserve $p^2 + m^2 = 0$

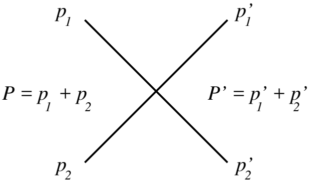
- $F^\mu p_\mu = 0$
- or $F^\mu + \partial^\mu m(x) = 0$

Without collisions: $f(p + F\Delta\tau, x + \frac{1}{m} p\Delta\tau) = f(p, x)$
Hence

$$\boxed{\left(p^\mu \partial_\mu + mF^\mu \frac{\partial}{\partial p^\mu}\right) \theta(p^0)\delta(p^2 + m^2) f(p, x) = 0}$$

where p^μ are independent in $f(p, x)$.

Particle flow in phase space with collisions



Described by scattering function:

$$W(p_1, p_2 | p_1', p_2') = s\sigma(s, \Theta)\delta(P' - P)$$

$$\cos\Theta = 1 + 2t/(s - 4m^2)$$
$$s = (p_1 + p_2)^2,\ t = (p_1 - p_1')^2$$
$$W(p_1, p_2 | p_1', p_2') = W(p_1', p_2' | p_1, p_2)$$

$R(p,x) d^4x \dfrac{\bar{d}^3 p}{2E}$ – Scatterings in which one of the initial particles has momentum p at space-time point x

$R'(p,x) d^4x \dfrac{\bar{d}^3 p}{2E}$ – Scatterings in which one of the final particles has momentum p at space-time point x

$$\boxed{p^\mu \partial_\mu f(p,x) = C[f]} = R'(p,x) - R(p,x)$$

Classical statistics:

$$R(p,x) = \int \frac{\bar{d}^3 p_2}{2E_2} \frac{\bar{d}^3 p_1'}{2E_1'} \frac{\bar{d}^3 p_2'}{2E_2'} f(p_1, x) f(p_2, x) W(p_1, p_2 | p_1', p_2')$$

Collision invariants and conservation laws

2-body collisions conserve

- Particle number
- Momentum

$$\left.\begin{array}{l}\\ \end{array}\right\}\text{Can show}\left\{\begin{array}{c}\psi(x) = a(x) + b_\mu(x)p^\mu \\[1em] \displaystyle\int \frac{\bar{d}^3p}{2E}\psi(x)C[f] = 0\end{array}\right.$$

for arbitrary $a(x)$, $b(x)$.

$\times$ both sides of $p^\mu\partial_\mu f(p,x) = C[f]$ by ψ and integrate over momentum space

$$b_\mu = 0 \implies \int \frac{\bar{d}^3p}{2E}p^\mu\partial_\mu f = 0 \implies \tfrac{1}{2}\partial_\mu \int \bar{d}^3p\,\tfrac{p^\mu}{E}f = 0 \quad\implies\quad \boxed{\partial_\mu j^\mu = 0}$$

$$a = 0 \implies \int \frac{\bar{d}^3p}{2E}p^\nu p^\mu\partial_\mu f = 0 \implies \tfrac{1}{2}\partial_\mu \int \bar{d}^3p\,p^\nu\,\tfrac{p^\mu}{E}f = 0 \quad\implies\quad \boxed{\partial_\mu T^{\mu\nu} = 0}$$

Equilibrium distribution (classical statistics)

Recall $p^\mu \partial_\mu f(p,x) = R'(p,x) - R(p,x)$ with:

$$R(p,x) = \int \frac{\bar{d}^3 p_2}{2E_2} \frac{\bar{d}^3 p'_1}{2E'_1} \frac{\bar{d}^3 p'_2}{2E'_2} f(p_1,x) f(p_2,x) W(p_1,p_2|p'_1,p'_2)$$

Note $\quad W(p_1,p_2|p'_1,p'_2) = W(p'_1,p'_2|p_1,p_2) \quad$ [time-reversal, parity]

Local equilibrium (vanishing collision term) is established if

$$f(p_1,x) f(p_2,x) = f(p'_1,x) f(p'_2,x) \quad \text{for } \textbf{all} \quad (p_a, p'_a)$$

Hence

$$\log f_1 + \log f_2 = \log f'_1 + \log f'_2 \quad \text{for } \textbf{all} \quad (p_a, p'_a)$$

$\log f_1 + \log f_2$ is a conserved quantity $\implies$ must be $\propto \psi(x) = a(x) + b_\mu(x) p^\mu$

$$f^{\rm eq}(p,x) = \exp[a(x) + b_\mu(x) p^\mu]$$

Identify: $a = \beta(x)\mu(x)$, $b_\mu = \beta(x) U_\mu(x)$
μ chemical potential – β inverse temperature – U^μ 4-velocity

Equilibrium distribution (quantum statistics)

With quantum statistics:

$$R(p,x) = \int \frac{\bar{d}^3 p_2}{2E_2} \frac{\bar{d}^3 p_1'}{2E_1'} \frac{\bar{d}^3 p_2'}{2E_2'} f_1 f_2 (1 \pm f_1)(1 \pm f_2) W(p_1, p_2 | p_1', p_2')$$

Bose enhancement
Fermi blocking

Local equilibrium (vanishing collision term) is established if

$$f_1 f_2 (1 \pm f_1')(1 \pm f_2') = f_1' f_2' (1 \pm f_1)(1 \pm f_2) \quad \text{for } \textbf{all} \quad (p_a, p_a')$$

Hence

$$\log f_1/(1 \pm f_1) + \log f_2/(1 \pm f_2) = \log f_1'/(1 \pm f_1') + \log f_2'/(1 \pm f_2') \quad \text{for } \textbf{all} \quad (p_a, p_a')$$

Now $\log f/(1 \pm f)$ is conserved quantity $\propto \psi(x) = a(x) + b_\mu(x) p^\mu$

$$f^{\mathrm{eq}}(p,x) = (\exp[a(x) + b_\mu(x) p^\mu \pm 1)^{-1}$$

Identify: $a = \beta(x)\mu(x)$, $b_\mu = \beta(x) U_\mu(x)$
μ chemical potential – β inverse temperature – U^μ 4-velocity

Fluid energy-momentum tensor

Distribution function for system in local equilibrium:

$$f^{\rm eq}(p,x) = \frac{1}{e^{\beta(U_\mu p^\mu - \mu)} \pm 1}$$

Energy-momentum tensor:

$$\begin{aligned} T^{\mu\nu} &= \int \frac{\bar{d}^3 p}{2E} 2p^\mu p^\nu f^{\rm eq}(p,x) \\ T^{\mu\nu} &= (e+p)U^\mu U^\nu + p g^{\mu\nu} \end{aligned}$$

where

$$\begin{aligned} e &= \int \bar{d}^3 p \, E \, f_0^{\rm eq}(p,x) \quad \text{rest frame energy density} \\ p &= \int \bar{d}^3 p \, \frac{\mathbf{p}^2}{3E} \, f_0^{\rm eq}(p,x) \quad \text{rest frame (kinetic) pressure} \end{aligned}$$