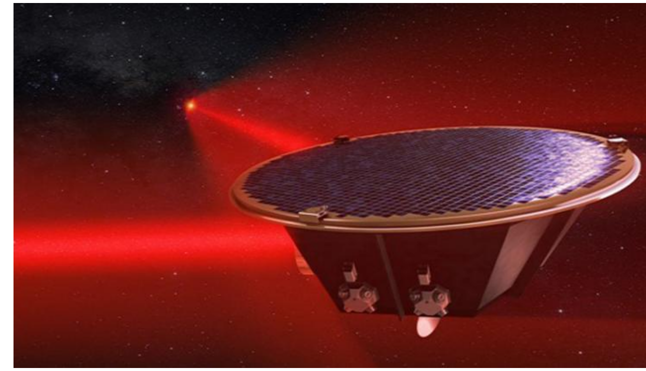


Gravitational waves from phase transitions in the early Universe

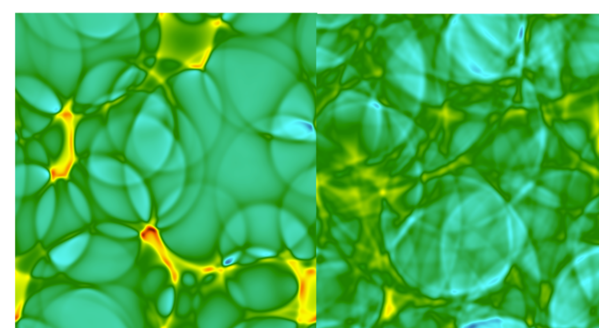
Mark Hindmarsh

Dept of Physics and Astronomy, University of Sussex

and

Helsinki Institute of Physics & Dept of Physics, University of Helsinki

Heraeus School
Heigenbrücken
21. syyskuuta 2018



Detection of gravity waves

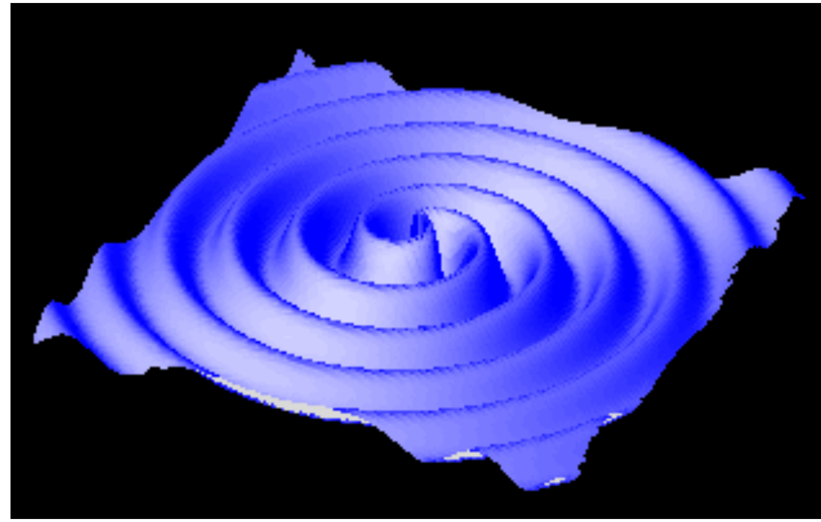
… in Brighton

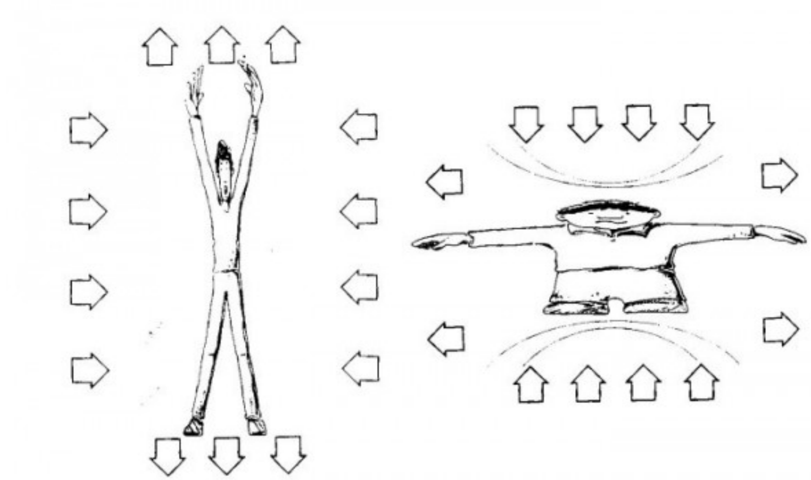
- Here they are!

- And now *gravitational* waves too!

Gravitational waves

- Predicted by Einstein, 1916
- Generated by accelerating, asymmetric mass-energy (quadrupole moment)
- Astrophysical sources: binary compact objects (**black holes**, **neutron stars**, white dwarfs); supernovae
- Cosmological sources: early universe





Detecting gravitational waves

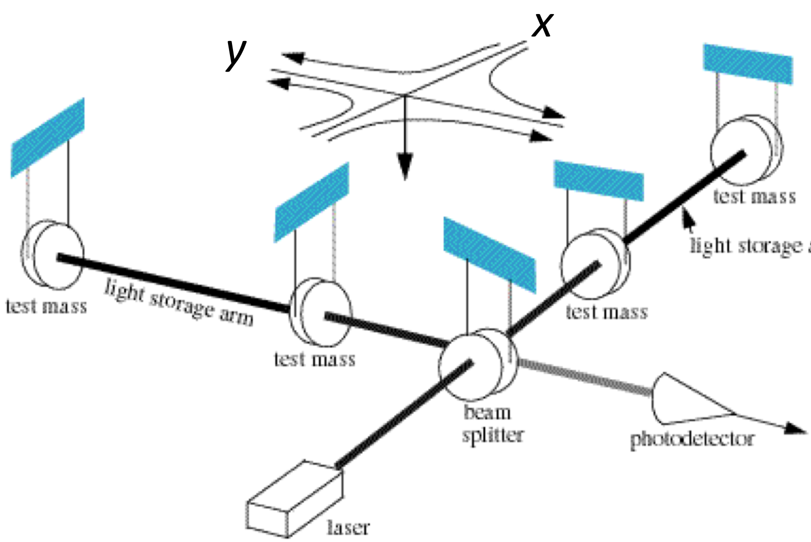
- Compare distances between test masses in two directions with laser interferometer
- Strain ≈ Metric perturbation

$$\frac{\Delta l}{l} = \frac{1}{2}(h_{xx} - h_{yy})$$

- Sensitivity: $\Delta l \approx 10^{-18}$ m @ 10^{2} Hz
-

Name	Location	Arm length
GEO	Germany	600m
(a)LIGO	USA (2)	4km
(a)VIRGO	Italy	3km
KAGRA	Japan	3km
LIGO-India	India	4km

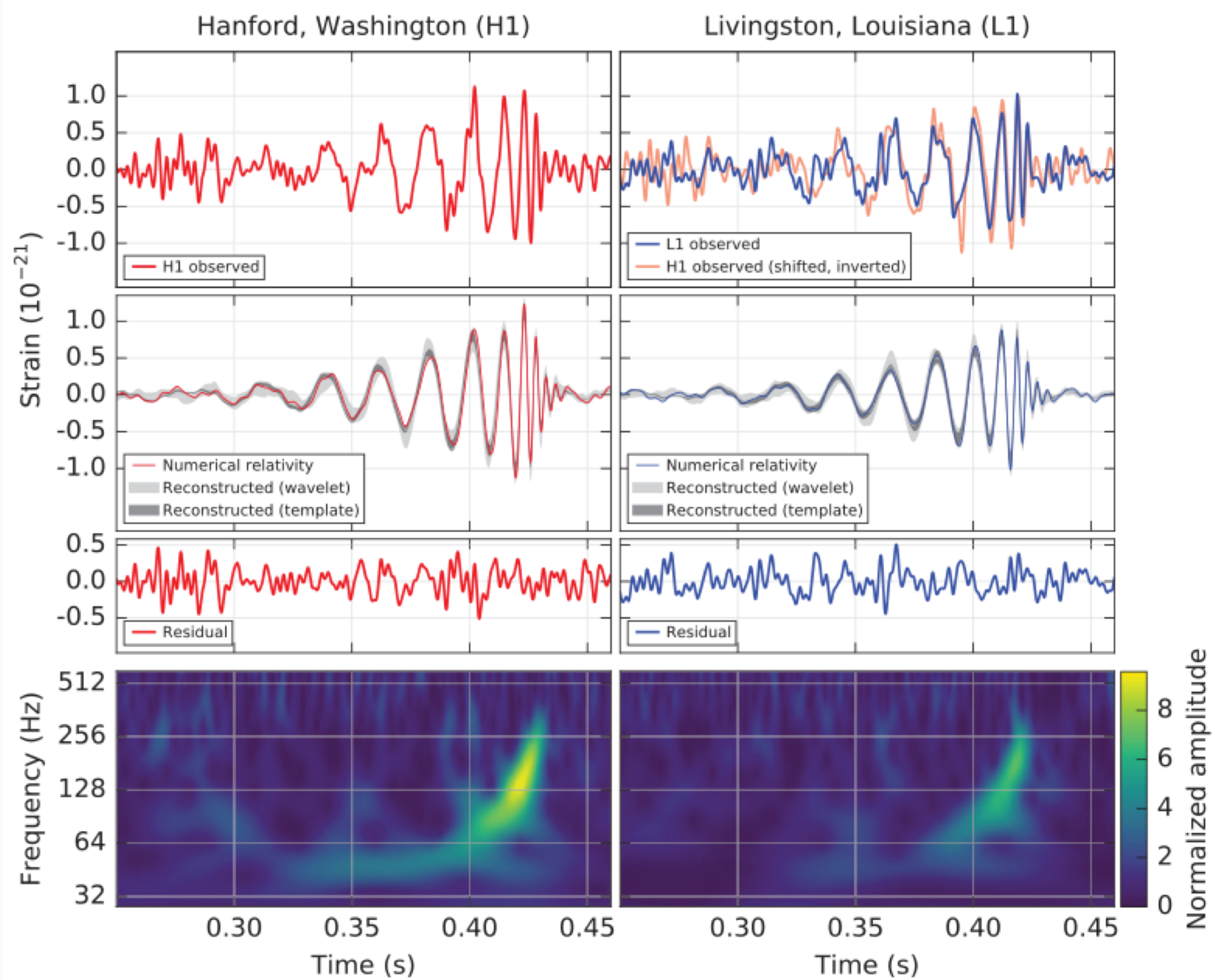
- Future: Einstein Telescope



LIGO

LIGO

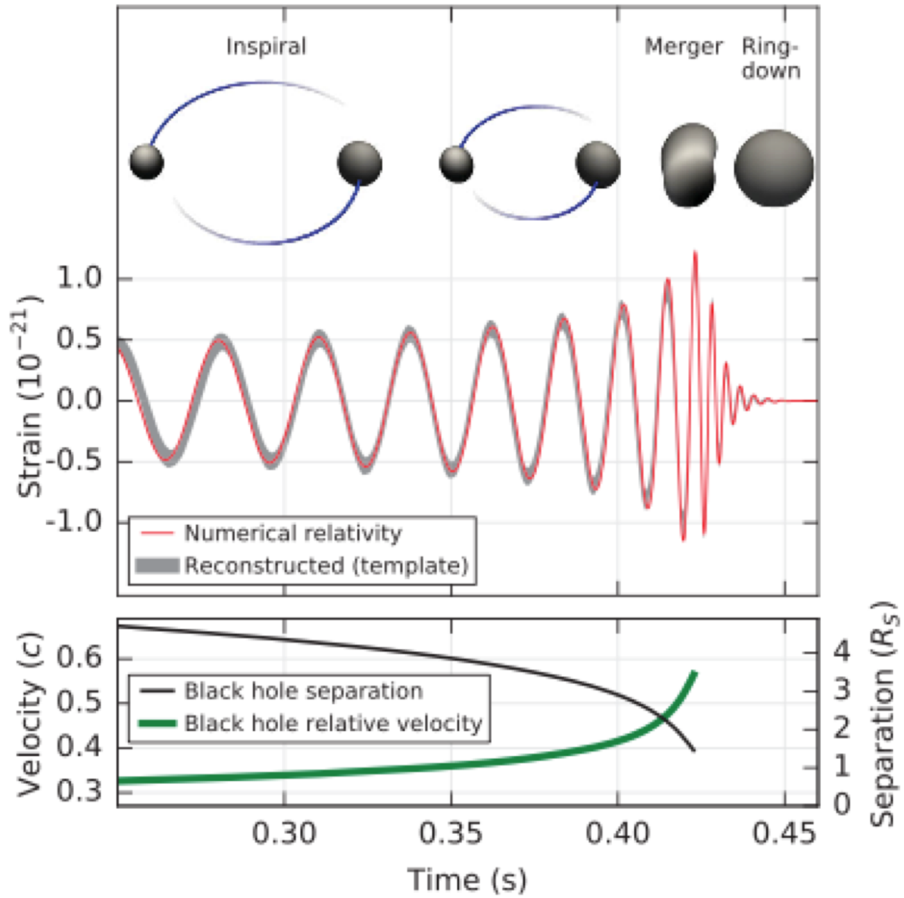
GW150914 strain and frequency spectrum



GW150914 binary black hole merger

- From inspiral waveform:
  - $M_1 = 36\pm5\ M_\odot$ $M_2 = 29\pm4\ M_\odot$
  - Distance ~ 400 Mpc
- Decay waveform: "ringdown" to a spinning BH

  $M_{final} = 62\pm4\ M_\odot$
- ~3 $M_\odot$ in gravitational waves



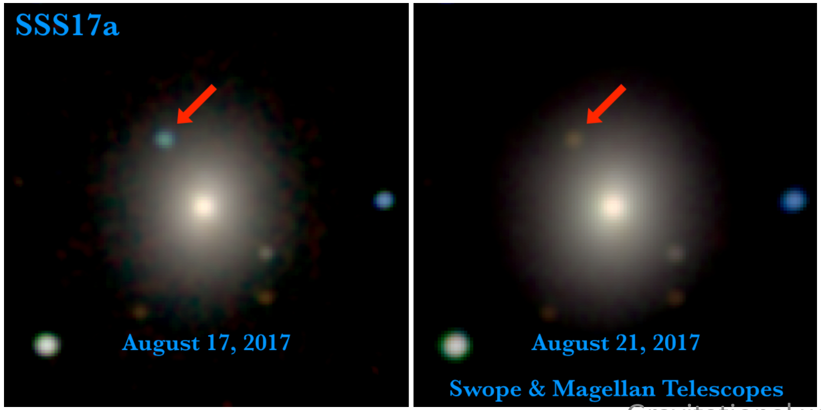
GW170817 binary NS merger

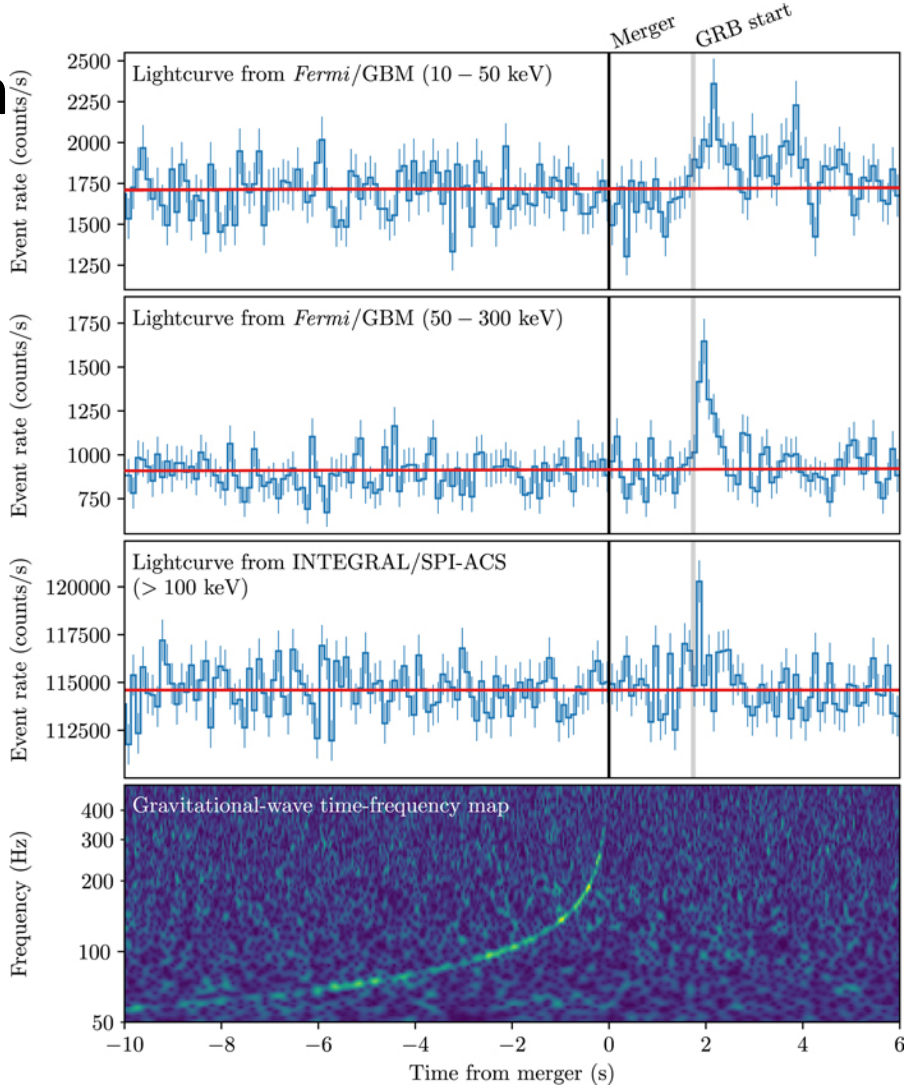
- 3 detectors - sky localisation
- Multiband detections
- GRB started ~1 s after:

$$|c_{\rm gw}^2 - 1| \lesssim 10^{-15}$$

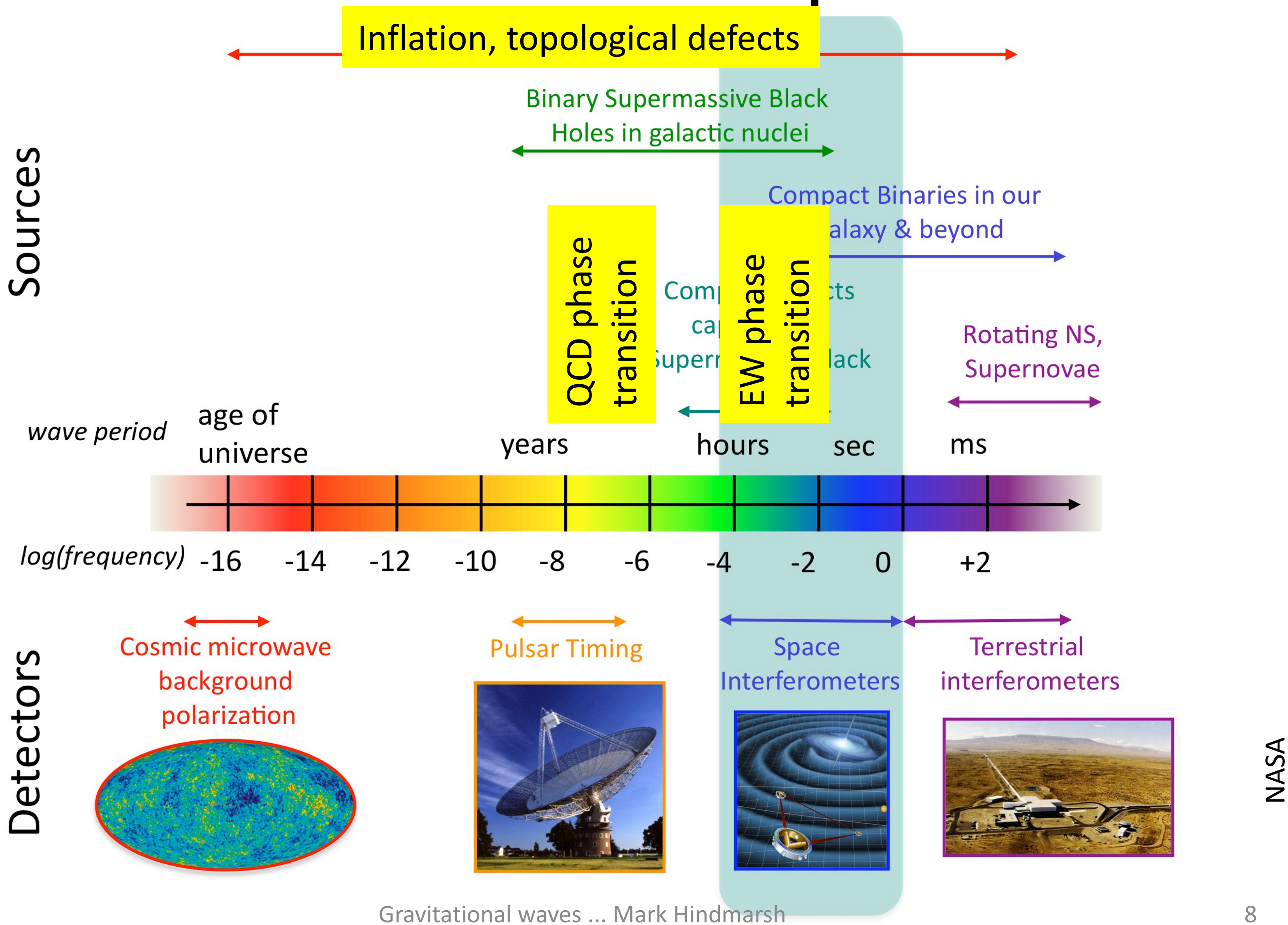
- Strong constraints on modifications of GR

Creminelli, Vernizzi; Wang et al; Baker et al 2017,





Gravitational wave spectrum



Gravitational waves from the early universe

- Events at time t generate waves with minimum frequency $f \approx 1/t$ (Hubble rate)
- Redshifted to a frequency now: $f_0 = (a(t)/a(t_0))f$
- Minimum frequencies (redshifted Hubble rates):

Event	Time/s	Temp/GeV	f_0/Hz
QCD transition	10^{-3}	0.1	10^{-8}
EW transition	10^{-11}	100	10^{-5}
?	10^{-25}	10^{9}	100
End of inflation	$\geq 10^{-36}$	$\leq 10^{16}$	$\leq 10^{8}$

Measures of gravitational waves

- Unit vectors along interferometer arms: l_i m_i
- Fourier transform of strain

$$\tilde{h}(f) = \frac{1}{2}\int_{-\infty}^{\infty} dt\, e^{-i2\pi f t} h_{ij}(l_i l_j - m_i m_j)$$

- One-sided power spectrum $S_h(f)$ ($f > 0$)

$$\langle \tilde{h}(f)\tilde{h}^*(f')\rangle = \frac{1}{2} S_h(f)\delta(f - f')$$

  - Characteristic strain (dimensionless) $h_c(f) = \sqrt{f S_h(f)}$
  - Root power spectral density ($\sqrt{\mathrm{Hz}^{-1}}$) $h(f) = \sqrt{S_h(f)}$
  - Energy density per logarithmic frequency interval:

$$\frac{d\rho_{\mathrm{gw}}}{d\ln f} = \frac{\pi}{G} f^3 S_h(f)$$

Measures of gravitational waves

- Unit vectors along interferometer arms: l_i m_i
- Fourier transform of strain

$$\tilde{h}(f) = \frac{1}{2}\int_{-\infty}^{\infty} dt\, e^{-i2\pi f t} h_{ij}(l_i l_j - m_i m_j)$$

- One-sided power spectrum $S_h(f)$ ($f > 0$)

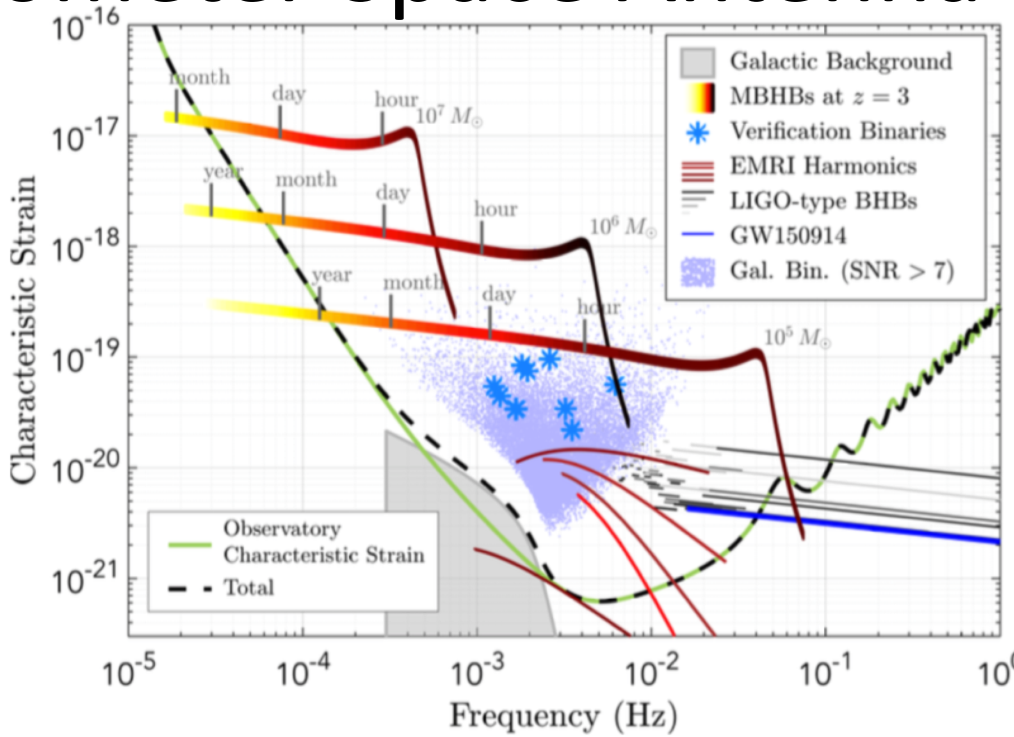
$$\langle \tilde{h}(f)\tilde{h}^*(f')\rangle = \frac{1}{2} S_h(f)\delta(f - f')$$

  - Characteristic strain (dimensionless) $h_c(f) = \sqrt{f S_h(f)}$
  - Root power spectral density ($\sqrt{\mathrm{Hz}^{-1}}$) $h(f) = \sqrt{S_h(f)}$
  - Fractional energy density per log frequency interval:

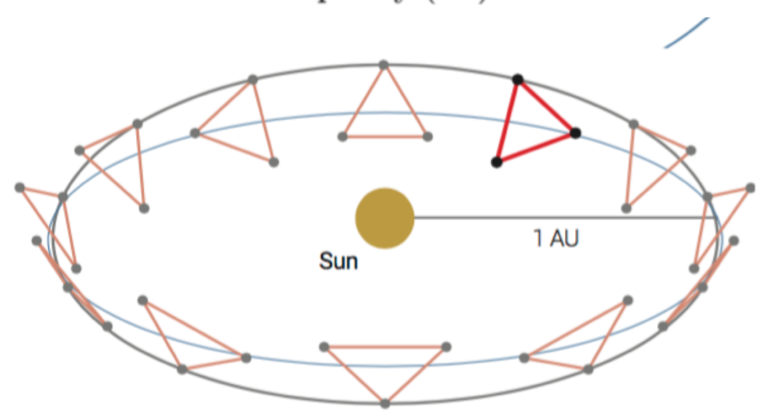
$$\frac{d\Omega_{\mathrm{gw}}}{d\ln f} = \frac{1}{\rho_{\mathrm{tot}}}\frac{d\rho_{\mathrm{gw}}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$

Laser Interferometer Space Antenna

- Launch by 2034
- 4-year mission (up to 10 years)
- 2.5M km arms
- Science objectives:
  - White dwarves
  - Black holes
  - Galaxy mergers
  - Extreme gravity
  - TeV-scale early Universe



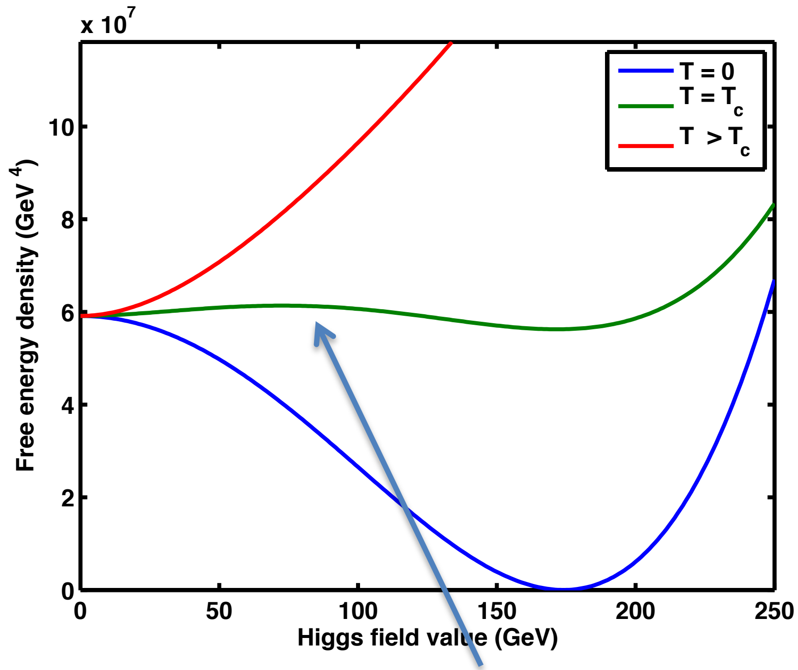
LISA sensitivity



LISA

Electroweak phase transition

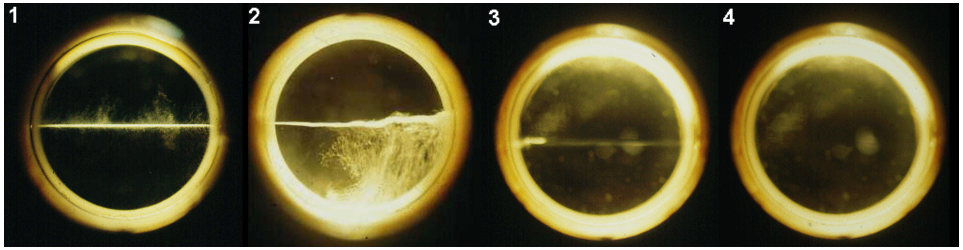
- Phase transition in weakly coupled gauge theories:
  (Kirzhnits 1972, Kirzhnits & Linde 1972)
- Free energy density of plasma depends on
  - Temperature *T*
  - Particle masses $m_i(\phi)$
- High *T*: reduce free energy by forcing Higgs ϕ to zero
- Electroweak phase transition: $T_c \approx v_{EW} \approx 100$ GeV (10^{15} K)
- High *T* (>> $m_i(\phi)$): $$V_T(\phi) = \frac{1}{2}A(T^2 - T_0^2)\phi^2 - \frac{1}{3}ET\phi^3 + \frac{1}{4}\phi^4$$

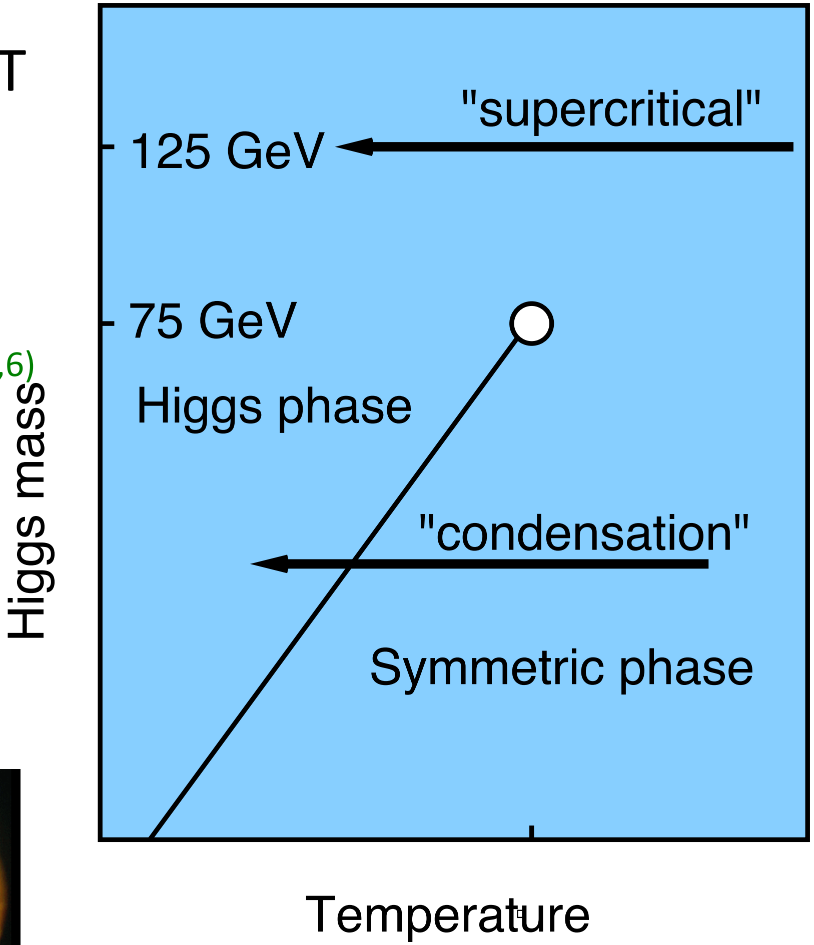


Potential barrier from cubic term in perturbative high-T expansion. First order transition?

Phase transitions: Standard Model

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction + effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2) on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over - a supercritical fluid





1^{st} order phase transitions Beyond the Standard Model

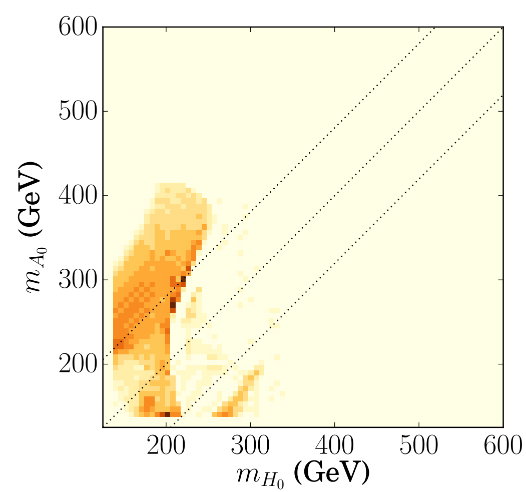
Departure from equilibrium

Kuzmin, Rubakov, Shaposhnikov (2009)

Baryogenesis

- SM + singlet
  - Strong 1^{st} order phase transitions possible
    Espinosa, Konstandin, Riva (2012)
    Cline, Kainulainen (2013)
    Kozaczuk (2015)
    Kotwal et al (2016)
  - Even without LHC traces
    Ashoorioon and Konstandin (2009)
    Curtin, Meade, Yu (2011)
  - *Zh* signal at future e^+e^-
    Curtin, Meade, Yu (2011)
    Cao, Huang, Xie, Zhang (2017)

- SM + Doublet ("2HDM")
  - Baryogenesis and B physics
    Cline, Kainulainen (2011)
  - Heavy A^0 ← strong 1^{st} order phase transition ($v(T_c)/T_c > 1$)
    Dorsch, Huber, No (2014)
  - Signal: $A^0 \to H^0 Z$
    Dorsch, Huber, Mimasu, No (2016)
    Andersen et al (2017)



tan(β) = 2.0, CP-conserving

1st order phase transitions Beyond the Standard Model

- SUSY with 1st order PT
  - NMSSM
  - nMSSM
  - GNMSSM

  Pietroni (1993)
  Davis, Froggatt, Moorhouse (1996)
  Cline, Kainulainen (1996)
  Huber, Schmidt (1999,2001)
  Panagiotakopoulos, Tamvakis (1999)
  Menon, Morrissey, Wagner (2004)
  Huang et al (2014)
  Kozaczuk et al (2015)

- TeV-scale strong dynamics
  - Minimal walking technicolor

    Järvinen, Kouvaris, Sannino (2010)

- Holographic models
  - Randall-Sundrum = SM + dilaton

  Creminelli, Nicolis, Rattazzi 2001
  Nardini, Quiros, Wultzer 2007
  Konstandin, Servant 2011

- String landscape

  Garcia, Krippendorf, March-Russell 2015

- SM + dim 6

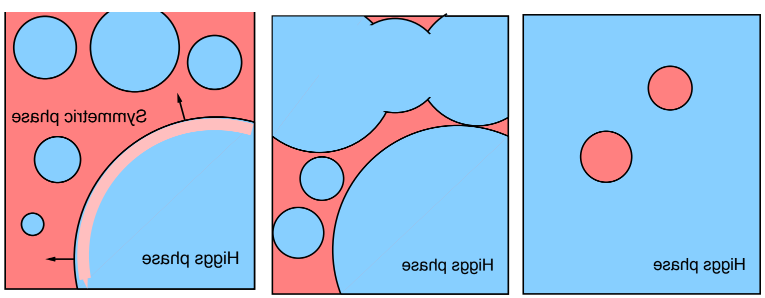
  $$V_{\rm eff}(H) = -\frac{\mu^2}{2}H^2 + \frac{\lambda}{4}H^4 + \frac{1}{8M^2}H^6$$

  - Tree level 1st order phase transition for $\lambda < 0$

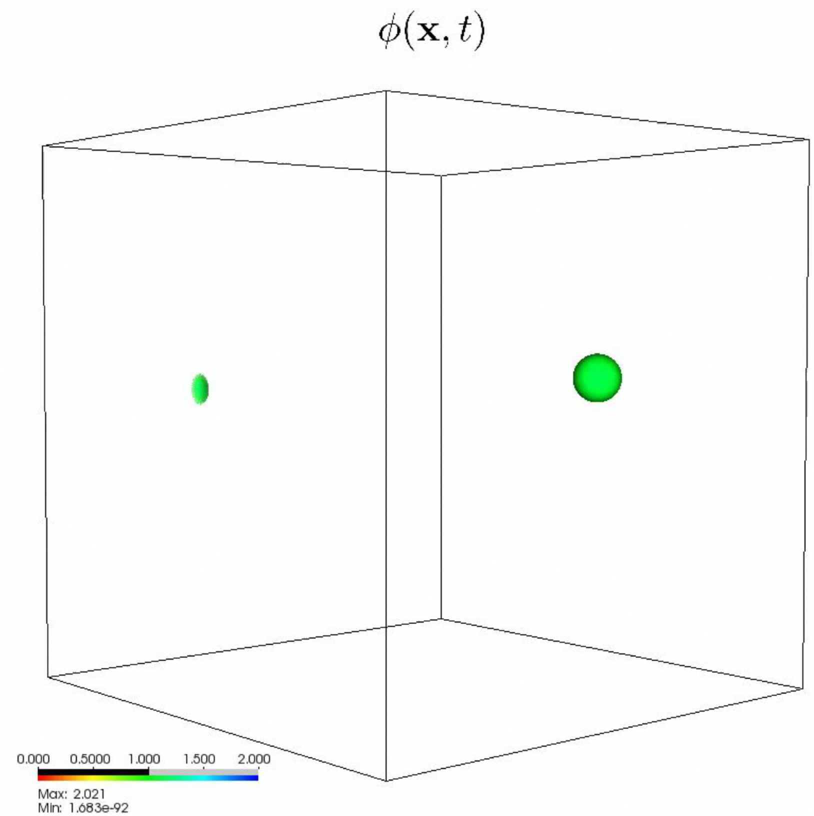
  Zhang (1993)
  Grojean, Servant, Wells (2004)
  Ham, Oh (2004)
  Bodeker et al (2004)
  Cao, Huang, Xie, Zhang (2017)

Little bangs in the Big Bang

- 1st order transition proceeds by nucleation of bubbles of Higgs phase
- Nucleation rate/volume *p*(*t*) rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Detectable gravitational waves?



Steinhardt (1982); Gyulassy et al (1984); Witten (1984); Enqvist et al (1992);

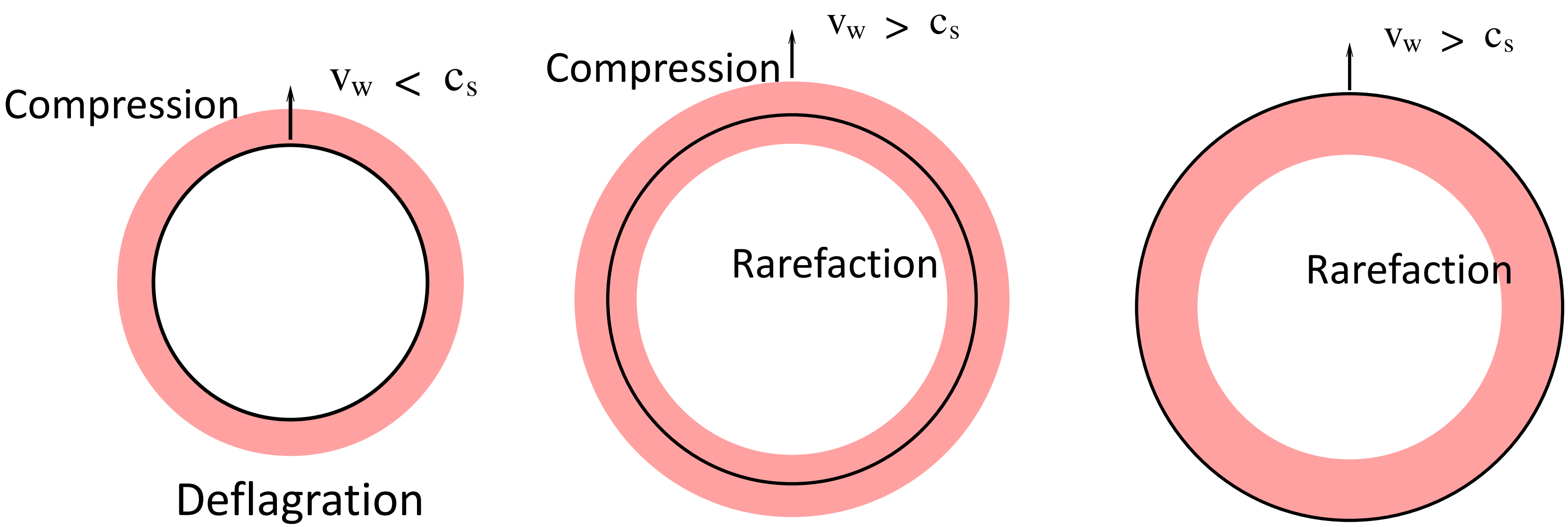


David Weir, Helsinki University

Scalar field

Hindmarsh, Huber, Rummukainen, Weir (2013)
Scalar only: Child, Giblin (2012)

Relativistic combustion



Deflagration

Supersonic deflagration
("hybrid")

Detonation

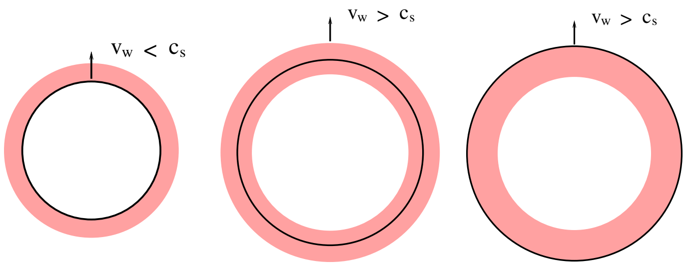
Landau & Lifshitz
Steinhardt (1984)
Kurki-Suonio, Laine (1991)
Espinosa et al (2010)

- Latent heat converted to kinetic energy, heat
- Wall velocity v_w from latent heat L and Higgs-plasma coupling $\eta(\phi)$
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$ from v_w and L
- Similarity solution $v(r/t)$, $w(r/t)$
- Runaway ($v_w \to 1$) not possible in gauge theory

Bodeker Moore 2017

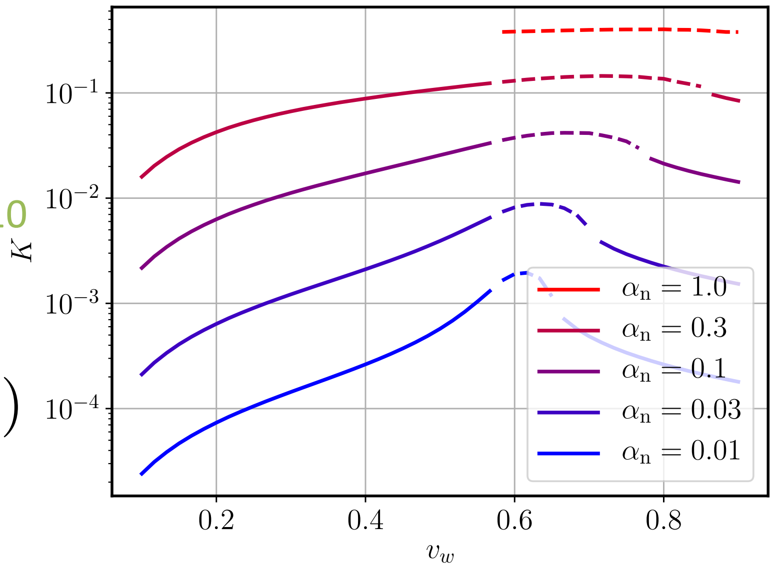
GWs from first order phase transitions

- Parametrise transition:
  - α = ("Latent heat")/(Total enthalpy)
  - β = transition rate (= - d log p / dt)
  - v_{w} = Bubble wall speed
  - H_* = Hubble rate at nucleation



- Derived parameters:
  - R_* = mean bubble separation (~v_{w}/β)
  - K = fluid kinetic energy fraction (depends on α, v_{w})

Steinhardt '84
Espinosa et al 2010



- Aim: GW power spectrum

$$\frac{d\Omega_{\mathrm{gw}}}{d\ln f} = \frac{1}{\rho_{\mathrm{tot}}}\frac{d\rho_{\mathrm{gw}}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$

Direct numerical simulation of an early universe phase transition

- Ingredients:

  Ignatius et al (1994), Kurki-Suonio, Laine (1996)

  – Higgs field $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$$
    - η coupling to fluid (models energy transfer, friction)
  – Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i P + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - E energy density, Z_i momentum density, V_i velocity, W γ-factor
  – Discretisation

  Wilson & Matthews (2003)
  Different approach: Giblin, Mertens (2013)

  – Metric perturbation

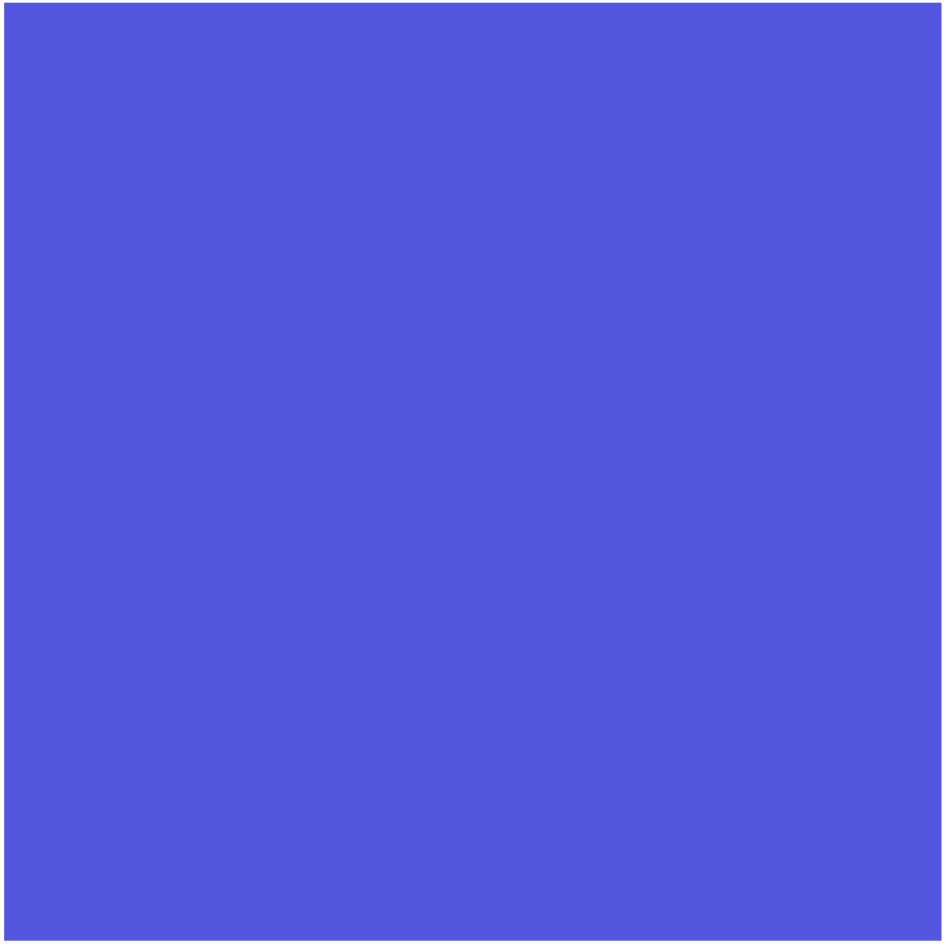
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

  Garcia-Bellido, Figueroa, Sastre (2008)

PRACE campaign

- Preparatory: 1M hrs CSC, Finland
- 2015/6: 17M CPU-hours
  Tier-0 (Hazel Hen, Stuttgart)
- 4200^3 lattice on 24k cores
- Wilson-Matthews fluid discretisation
- Simultaneous bubble nucleation

Hindmarsh, Huber, Rummukainen, Weir 2017



Kinetic energy in slice (1200^3)

GW energy density

$$\rho_{\rm GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}^2 \rangle.$$

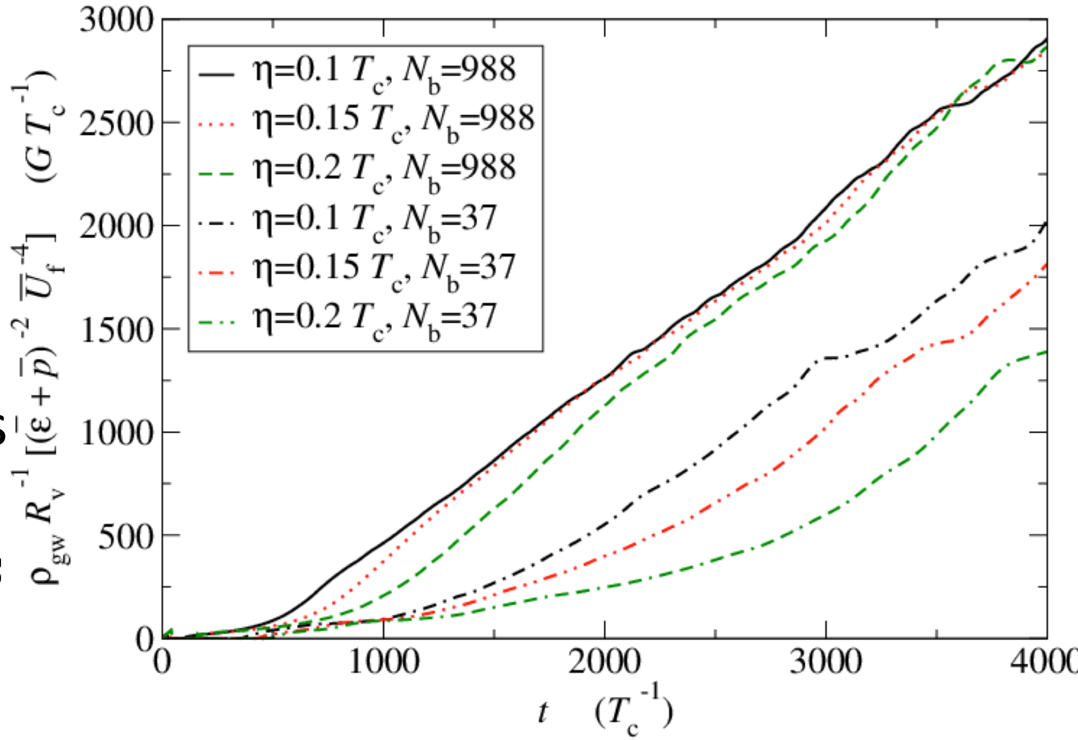
- Fluid sources GWs continuously:

$$T_{ij} = (\bar\epsilon + \bar p) V_i V_j \big|^{TT}$$

- GW energy density grows with time

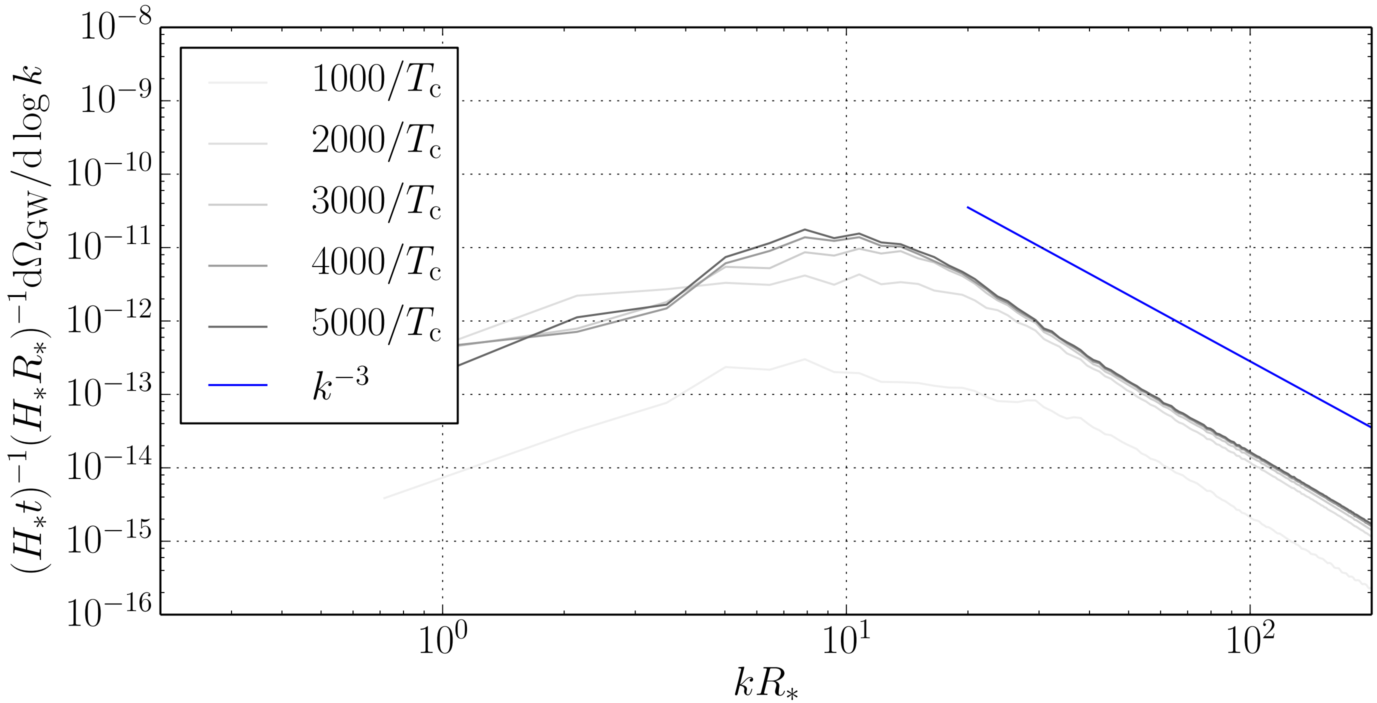
$$\rho_{\rm GW} \propto t\, G L_{\rm f} [(\bar\epsilon + \bar p)\bar U_{\rm f}^2]^2$$

- GW energy density depends on
  - Lifetime of source τ
  - Flow length scale $L_{\rm f} \sim R_*$
  - Kinetic energy fraction K



Hindmarsh, Huber, Rummukainen, Weir (2015)

GW power spectra: detonation

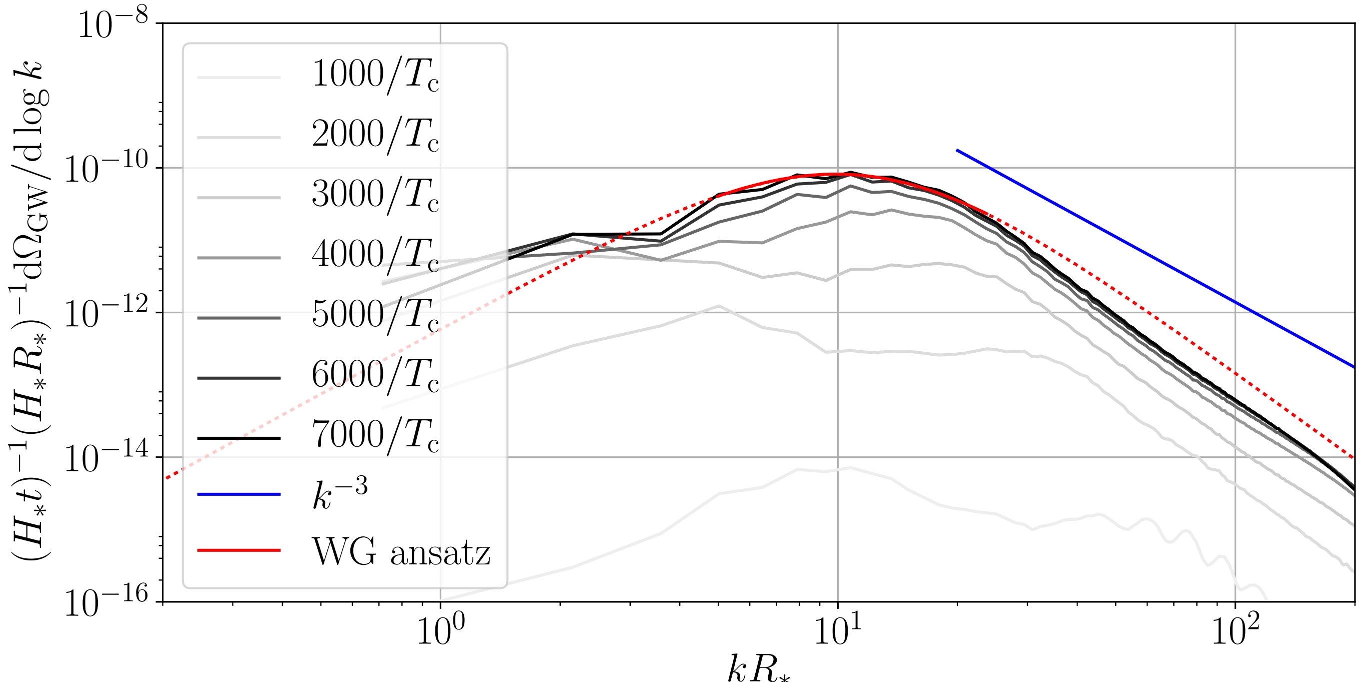


- Transition strength: $\alpha = 0.01$
- Wall speed: $v_w = 0.92$
- Mean bubble separation: $R_* = 1900/T_c$

- Peak at $kR_* \sim 10$
- Approx k^{-3} spectrum at high k

Hindmarsh, Huber, Rummukainen, Weir (2017)

GW power spectra: deflagration

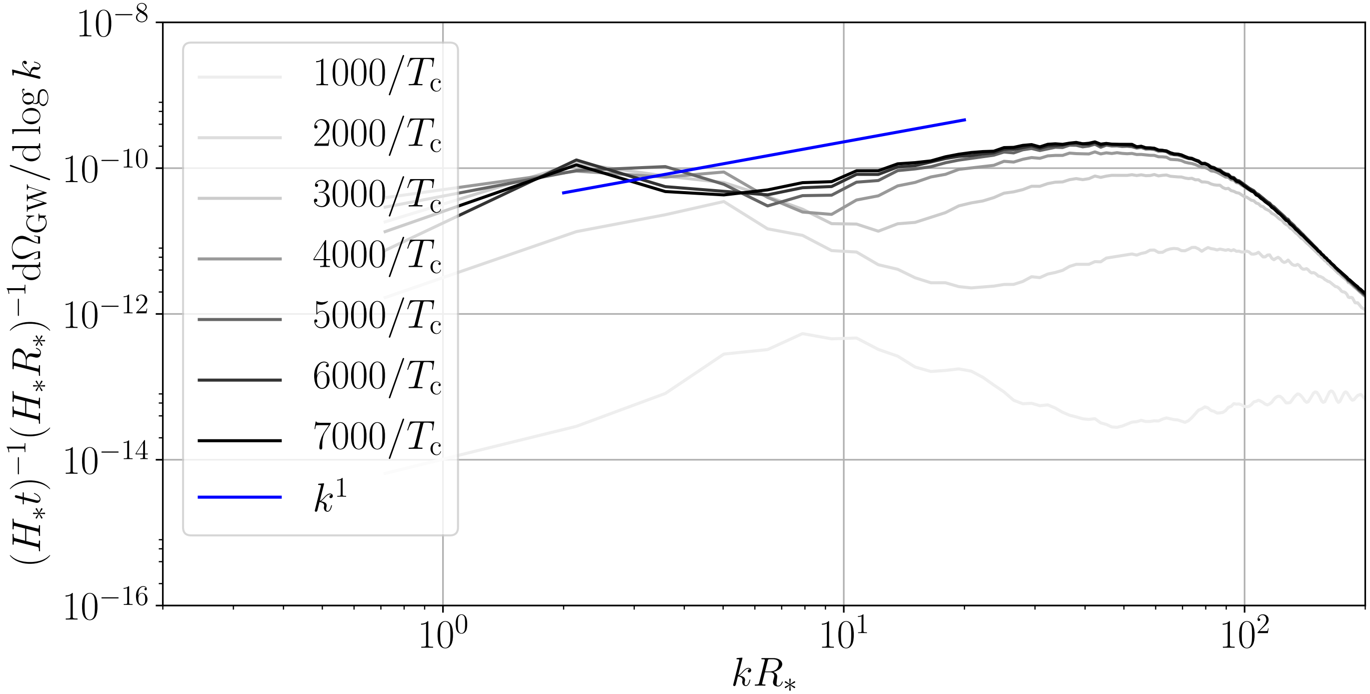


- Transition strength: $\alpha = 0.01$
- Wall speed: $v_w = 0.44$
- Mean bubble separation: $R_* = 1900/T_c$

- Peak at $kR_* \sim 10$
- Approx k^{-4} spectrum at high k

Hindmarsh, Huber, Rummukainen, Weir (2017)

GW power spectra: near c_s



- Transition strength: $\alpha = 0.01$
- Wall speed: $v_w = 0.56$
- Mean bubble separation: $R_* = 1900/T_c$

- Peak at $kR_* \sim 40$
- Approx k spectrum at lower k
- Peak length scale from sound shell thickness

Hindmarsh, Huber, Rummukainen, Weir (2017)

GWs from phase transitions

- Gravitational waves generated by shear stress fluctuations
- Shear stress ~ kinetic energy
- Kinetic energy from latent heat
- Phase transition parameters:
  - α = (Latent heat)/(Total energy)
  - $K(\alpha,\beta,v_w)$ = fluid kinetic energy fraction
  - Timescales τ_a and τ_b
    - Duration of stresses
    - Coherence time of stress fluctuations

$$\Omega_{\rm GW} \sim \frac{1}{G\rho} \left\langle \left| \dot{h}_{ij}(t) \right|^2 \right\rangle$$

$$\dot{h}_{ij} \sim G \int dt' \cos[k(t-t')] T^{TT}_{ij}(k,t')$$

$$T_{ij} \sim \rho U_i U_j$$

$$\dot{h} \sim \tau (G\rho) K$$

$$\Omega_{\rm GW} \sim \frac{\tau_a \tau_b}{G\rho} (G\rho)^2 K^2$$

$$\Omega_{\rm GW} \sim (H_* \tau_a)(H_* \tau_b) K^2$$

$$\Omega_{\rm GW,0} \sim \Omega_{\rm rad,0} (H_* \tau_a)(H_* \tau_b) K^2$$

Lifetime of sound waves

- Sound damped by viscosity

$$\left(\frac{4}{3}\eta_{\rm s} + \zeta\right) \nabla^2 V_\parallel^i$$

- Shear viscosity $\eta_{\rm s} \sim T^3/e^4 \ln(1/e)$
- Lifetime of scale *R*: $\tau_\eta(R) \sim e^4 \ln(1/e) R^2 T.$
- Longer than Hubble time for scales

$$R \gg \frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{m_{\rm Pl} e^4}\right) \sim 10^{-11} \frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{100 \text{ GeV}}\right),$$

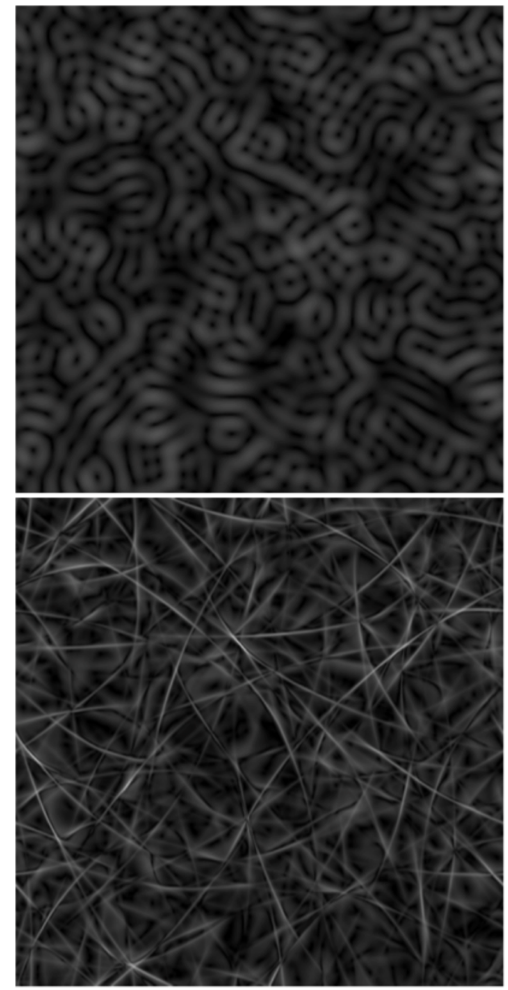
- Effective lifetime is Hubble time, unless …
- … shocks (& turbulence) develop, at $\tau_{\rm s} \sim R_*/\bar{U}_{\rm f}$
- Pure acoustic waves only if $\bar{U}_{\rm f} < R_* H_*$

Turbulence

- Shocks develop after $\tau_{\rm s} \sim R_*/\bar{U}_{\rm f}$
- Shocks become turbulent, turbulence decays after $\tau_{\rm tu} \sim R_*/\bar{U}_{\rm f}$
- Autocorrelation time (eddy turn-over time) $\tau_{\rm tu} \sim R_*/\bar{U}_{\rm f}$
- Estimate GW power

$$\Omega_{\rm GW}^{\rm tu} \sim (H_* R_*/\bar{U}_{\rm f})^2 K^2 \sim (H_* R_*)^2 K$$

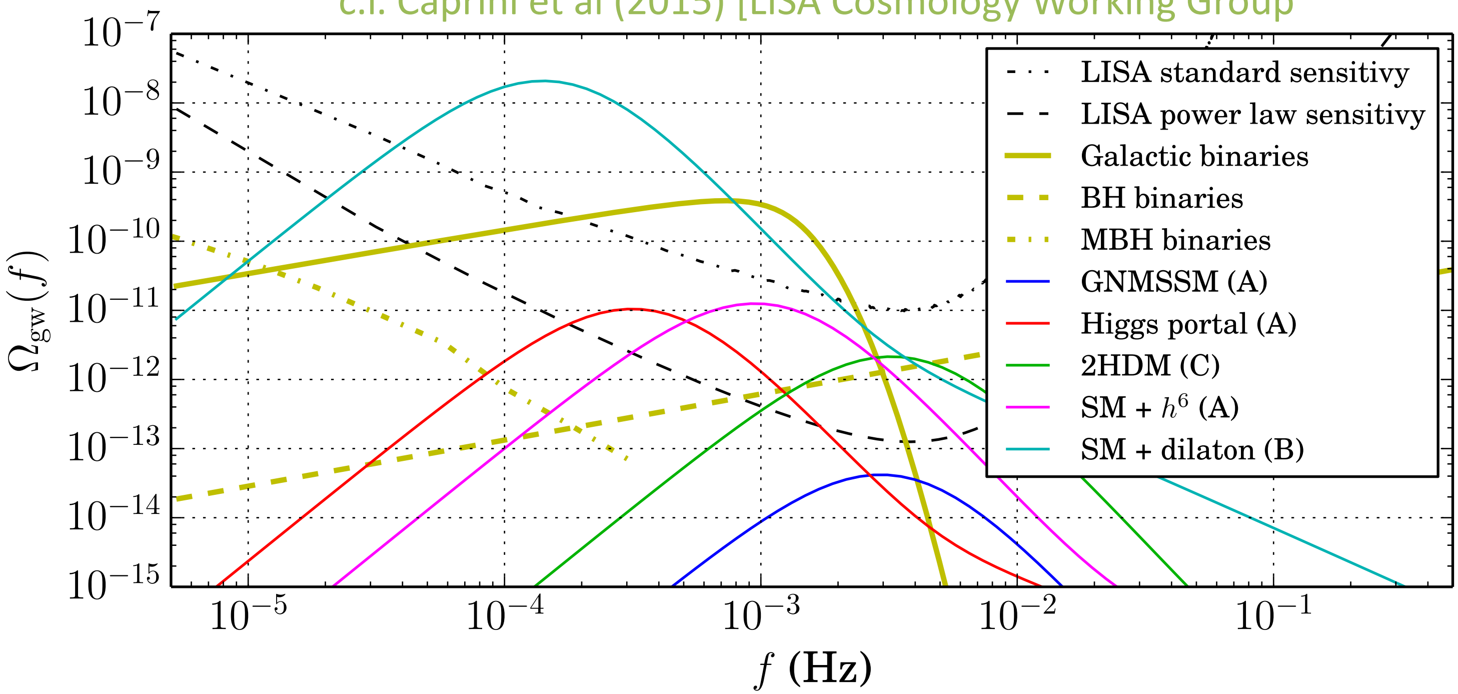
- Less than acoustic GW power if $\tau_{\rm s} \gg H_*^{-1}$
- Disagreement about spectrum:
  - $f^{-2/3}$ Caprini, Durrer, Servant 2009
  - $f^{-9/2}$ Kamionkowsky, Kosowsky, Turner 1994; Gogoberidze, Kahniashvili, Kosowsky 2007
- Turbulence: magnetic field dynamo



Pen, Turok 2015

LISA prospects for EW phase transition

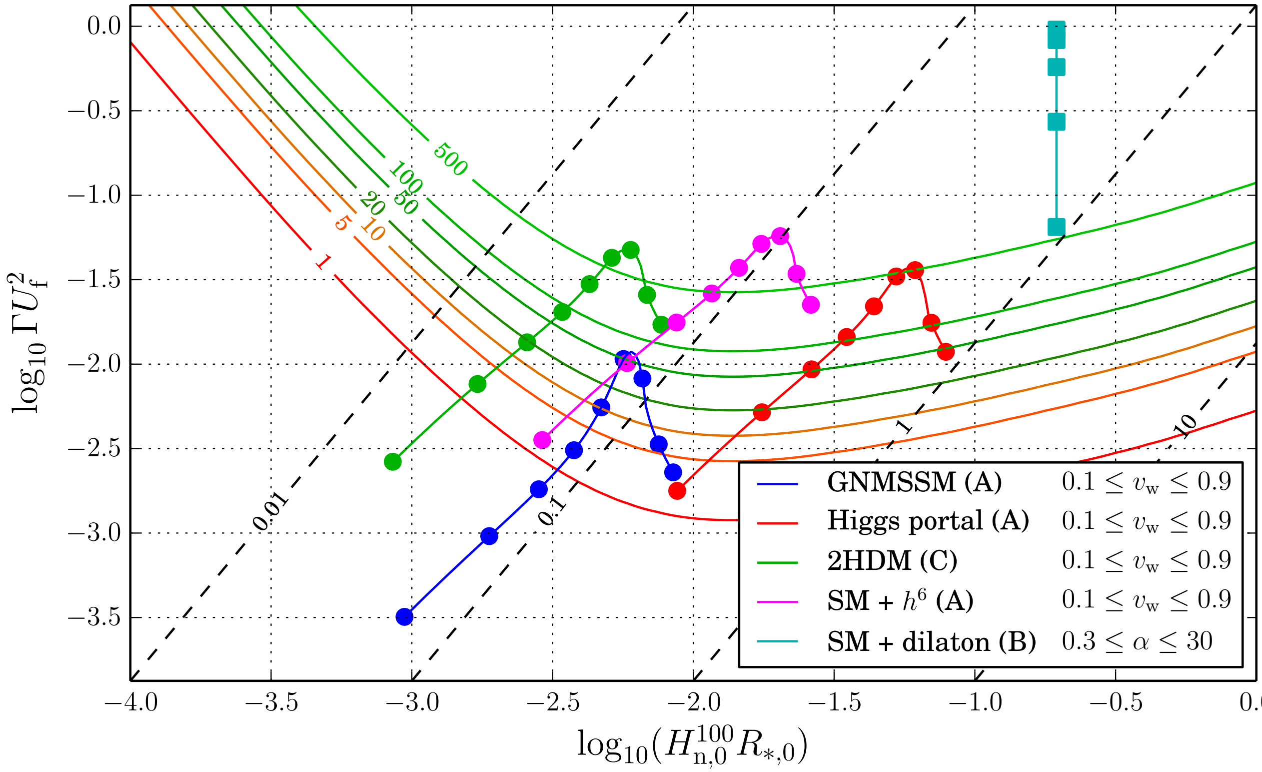
c.f. Caprini et al (2015) [LISA Cosmology Working Group



Model	T_n/GeV	α	β/H_n	v_w
GNMSSM A	112	0.037	277	0.95
Higgs portal A	70.6	0.09	47.35	0.95
2HDM C	51.6	0.111	663	0.95
SM + h^6 A	63	0.13	160	0.95
SM + dilaton B	100	1	15	0.95

LISA GW detection prospects

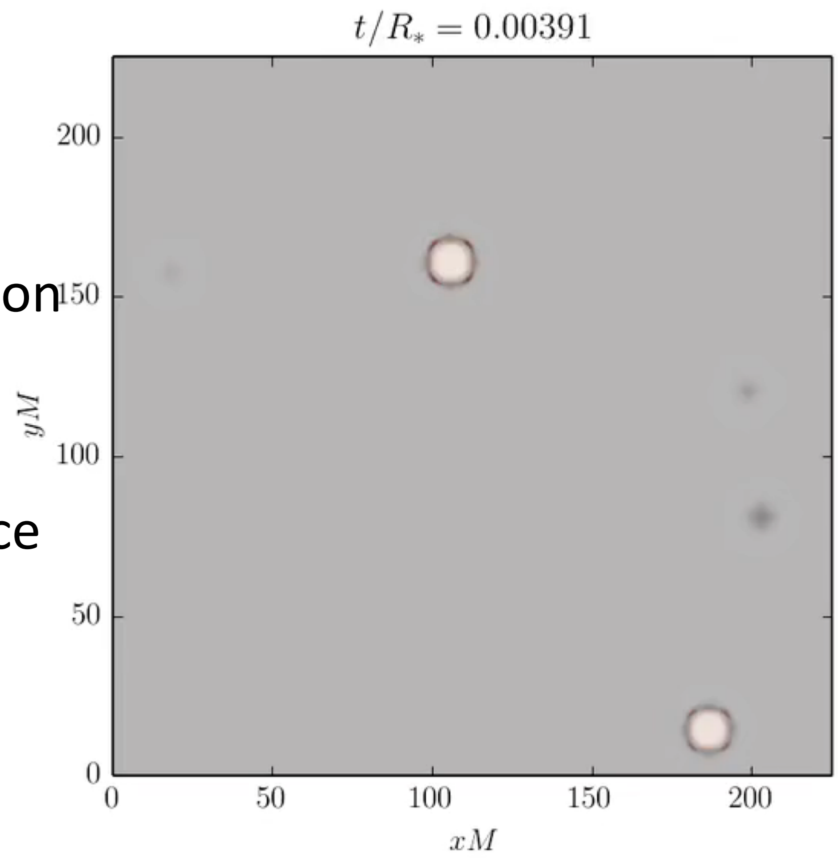
c.f. Caprini et al (2016) [LISA Cosmology Working Group



- Signal-to-noise ratio SNR; (eddy turn-over time)/(Hubble time)
- Predictions sensitive to parameters (α,β,v_w)
- Significant uncertainties in parameters, particularly v_w
- Turbulent signal underestimated

Gravitational waves from a vacuum phase transition

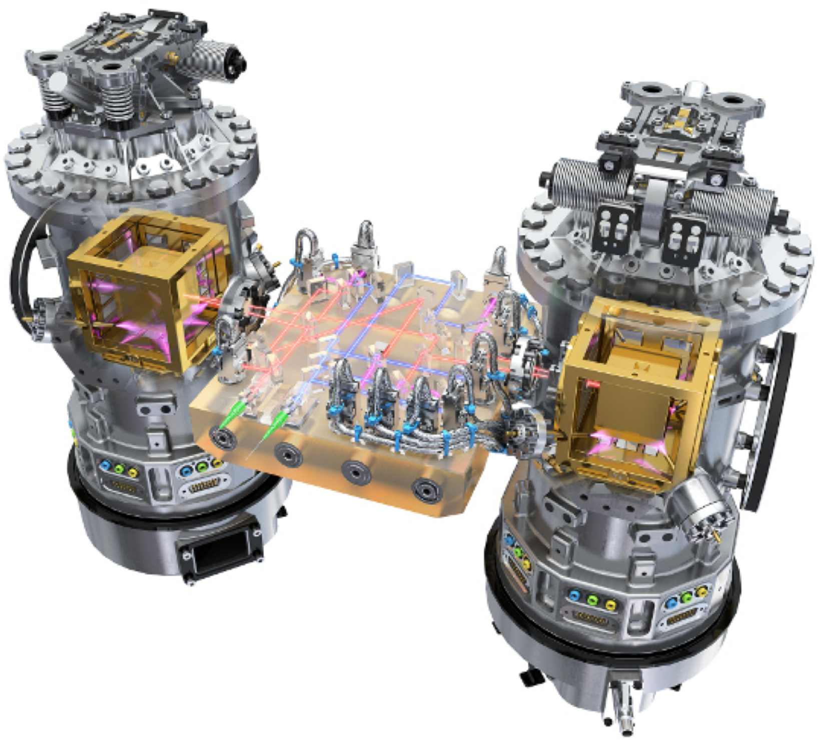
- Extreme supercooling: energy-momentum dominated by scalar field
- Sub-dominant or negligible contribution from fluid
- Bubble walls accelerate: $v_w \to 1$
- Negligible sound waves and turbulence
- Gravitational waves energy density shown in red

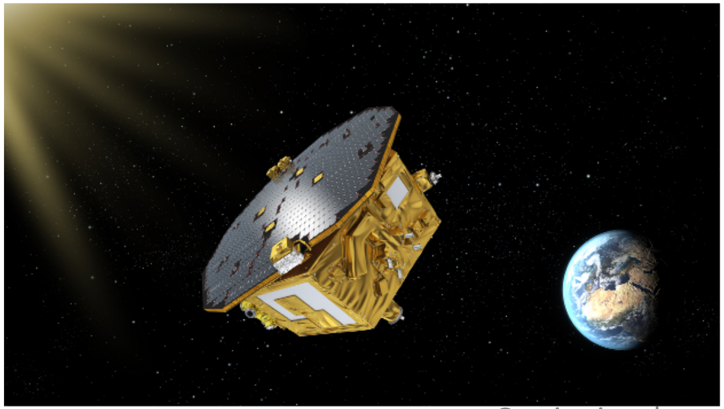


Cutting, Hindmarsh, Weir 2018

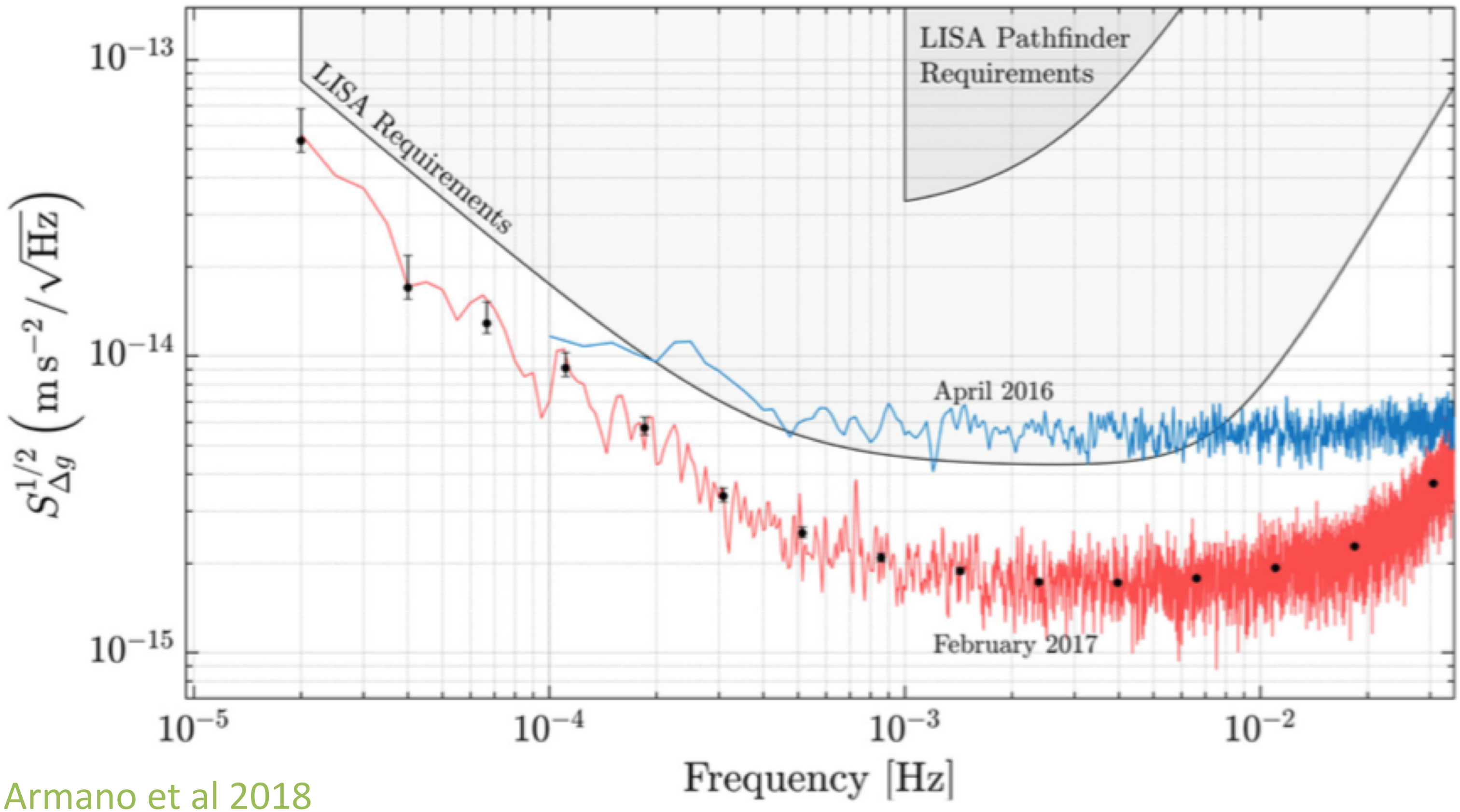
LISA Pathfinder

- Technology tester for LISA
- Test masses in free-fall at L1
- Control and measure motion
- Launched 3/12/15
- Masses released 3/2/16
- Mission end 30/6/17
- Quantified acceleration noise – how good is the free fall





LISA Pathfinder acceleration noise



Armano et al 2018

Geodesic deviation equation relates acceleration noise to strain noise

Acceleration noise 10^{-15} m s^{-2} Hz$^{-1/2}$ = characteristic strain ~ 10^{-21} at $f = 10^{-2}$ Hz

Summary

- GWs probe of physics of early universe
- 1st order phase transitions in the early universe produce gravitational waves
  - Source: sound waves from the nucleating droplets of the low temperature phase
  - Loud sound may become turbulent
  - GW spectrum contains information about phase transition parameters
  - Correlated collider signals
- LISA 2034
  - LISA Pathfinder success
  - Could detect a BSM electroweak phase transition at t = 10 picoseconds