Kaon Decays

more than 50 decay channels in the SM

$\rightarrow$ fertile field for CHPT

Two distinct classes of weak decays

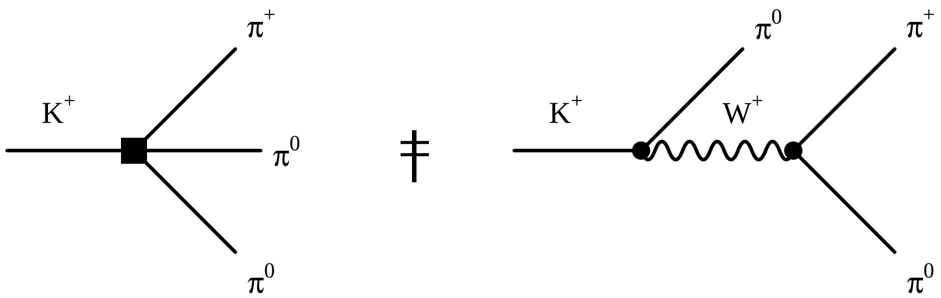
semileptonic		nonleptonic
attached	leptons	not attached
	to W boson	
not connected	hadrons	connected
	by W boson	

Chiral Lagrangian

strong	interactions	weak
$\sum_n \mathcal{L}_n^{\text{strong}}$	$n = 2, 4, 6, \ldots$	$\sum_n \mathcal{L}_n^{\Delta S=1}$

Beware : $K \to \pi l^+ l^-$ are nonleptonic decays according to this classification

Example : $K \to 3\pi$



Semileptonic Decays

all realistic decays calculated to one-loop accuracy

$\to$ in principle completely determined

in terms of LECs L_i

calculations to $O(p^6)$: $K \to \pi l \nu_l$ Post, Schilcher

$K \to \pi\pi l \nu_l$ Amoros et al.

still hampered by poorly known LECs of $O(p^6)$

Theoretical and experimental status of 1994 :

2nd DAΦNE Handbook of Physics

$K \to l\nu_l\gamma$

$$A(K(p) \to l\nu_l\gamma(q)) = A_{\rm IB} + A_{\rm SD}$$

$A_{\rm IB}$: given in terms of $A(K \to l\nu_l)$

(Bremsstrahlung)

structure dependent amplitude :

$$\begin{aligned} A_{\rm SD} &= \frac{eG_F}{\sqrt{2}} V_{\rm us}^* \varepsilon_\mu^* \overline{u}\gamma_\nu(1-\gamma_5)v \times \\ &\quad \left[F_V(W^2)\varepsilon^{\mu\nu\rho\sigma}q_\rho p_\sigma - F_A(W^2)(pqg^{\mu\nu} - p^\mu q^\nu)\right] \\ W^2 &= (p-q)^2 \end{aligned}$$

$O(p^2)$: $F_V = F_A = 0$

$O(p^4)$: constant form factors

$$F_V = \frac{\sqrt{2}M_K}{8\pi^2 F_\pi} , \qquad F_A = \frac{\sqrt{2}M_K}{F_\pi}(L_9 + L_{10})$$

$\to$ CHPT prediction (Donoghue, Holstein) :

$$\begin{aligned} F_V + F_A &= 0.14 \\ F_V - F_A &= 0.05 \end{aligned}$$

Heintze et al. (1979) $(l = e)$:

$$F_V + F_A = 0.148 \pm 0.010$$

new exp. result for $l = \mu$ (BNL-E787) :

$$F_V + F_A = 0.165 \pm 0.013$$
$$F_V - F_A = 0.102 \pm 0.085$$

- consistency check for CHPT at $O(p^4)$
- exp. has reached $O(p^6)$sensitivity

$K \to l\nu_l l^+ l^-$

form factors : 3 axial (dominant)

1 vector

Remark : $e\nu_e$ modes strongly helicity suppressed at $O(p^2)$(Bremsstrahlung)

branching ratios

(electronic modes: $m_{e^+e^-} \geq 140$ MeV)

$K^+ \to$	$\mu^+\nu_\mu e^+ e^-$	$e^+\nu_e e^+ e^-$	$\mu^+\nu_\mu \mu^+ \mu^-$
tree	$5.0 \cdot 10^{-8}$	$2.1 \cdot 10^{-12}$	$3.8 \cdot 10^{-9}$
1-loop	$8.5 \cdot 10^{-8}$	$3.4 \cdot 10^{-8}$	$1.35 \cdot 10^{-8}$
exp.	$(1.23 \pm 0.32) \cdot 10^{-7}$	$(2.8^{+2.8}_{-1.4}) \cdot 10^{-8}$	$\leq 4.1 \cdot 10^{-7}$

$K \to \pi l \nu_l \gamma$

10 form factors :
complicated structure (Bijnens, E., Gasser)
rates dominated by Bremsstrahlung

Example : $K_L \to \pi^\pm e^\mp \nu_e \gamma$

branching ratio with cuts

$$E_\gamma \geq 30 \text{ MeV} \qquad \Theta_{e\gamma} \geq 20^o$$

	$BR(K_L \to \pi^\pm e^\mp \nu_e \gamma)$
Bremsstrahlung	$3.6 \cdot 10^{-3}$
$O(p^4)(L_i$ only)	$4.0 \cdot 10^{-3}$
$O(p^4)$total (L_i and loops)	$3.8 \cdot 10^{-3}$
experiment (NA31)	$(3.61 \pm 0.14 \pm^{0.21}_{0.15}) \cdot 10^{-3}$

K_{l3} and K_{l4} decays

$O(p^6)$ $(L \leq 2)$ calculations finished
Amoros, Bijnens, Talavera; Post, Schilcher

phenomenological analysis forthcoming

Nonleptonic K Decays

Recall :

NEEDED

Effective Lagrangian for an effective Hamiltonian

From the Fermi scale to the kaon mass

M_W	$SU(3) \times SU(2) \times U(1)$ SM	quarks, gluons, leptons gauge bosons, Higgs
OPE	$\Downarrow$	perturbative
m_c	$\mathcal{L}_{\rm QCD}^{N_f=3}, \mathcal{L}_{\rm eff}^{\Delta S=1}, \ldots$	light quarks, gluons leptons, photon
symmetries	$\Downarrow$	nonperturbative
M_K	CHPT	hadrons leptons, photon

Effective Lagrangian $\mathcal{L}_{\rm eff}^{\Delta S=1}$

without QCD corrections ($C_+ = C_- = 1$)

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\Delta S=1} &= -\frac{4G_F}{\sqrt{2}} V_{ud}V_{us}^* \overline{s_L}\gamma^\mu u_L \overline{u_L}\gamma_\mu d_L \\
&= -\frac{2G_F}{\sqrt{2}} V_{ud}V_{us}^* (C_+Q_+ + C_-Q_-) \\
Q_\pm &= \overline{s_L}\gamma^\mu u_L \overline{u_L}\gamma_\mu d_L \pm \overline{s_L}\gamma^\mu d_L \overline{u_L}\gamma_\mu u_L
\end{aligned}$$

Q_+ : 27-plet $\qquad Q_-$: octet

with QCD corrections $\rightarrow$

short-distance enhancement (suppression)
of Q_- (Q_+)

at leading order in α_s :
$C_-(1 \text{ GeV}) \simeq 2.2 \qquad C_+(1 \text{ GeV}) \simeq 0.7$

$\rightarrow$ not enough to understand the

$\Delta I = 1/2$ rule (octet enhancement)

additional effects : penguin operators
final state interactions

$K \to \pi\pi$

isospin decomposition

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= A_{1/2} e^{i\delta_0^0} + \frac{1}{\sqrt{2}}(A_{3/2} + A_{5/2}) e^{i\delta_0^2} \\
A(K^0 \to \pi^0\pi^0) &= A_{1/2} e^{i\delta_0^0} - \sqrt{2}(A_{3/2} + A_{5/2}) e^{i\delta_0^2} \\
A(K^+ \to \pi^+\pi^0) &= (\frac{3}{2} A_{3/2} - A_{5/2}) e^{i\delta_0^2}
\end{aligned}
$$

$\delta^I_{l=0}(M_K^2)$: $\pi\pi$ phase shifts at $s = M_K^2$

SM (isospin limit) :

$$A_{5/2} = 0$$

alternative notation (esp. for K^0 decays)

$$A_0 = A_{1/2} \; , \qquad A_2 = A_{3/2} + A_{5/2}$$

Lowest-order $\Delta S = 1$ chiral Lagrangian

Recall :

$$\mathcal{L}_{\rm eff}^{\Delta S=1} \sim (8_L, 1_R) + (27_L, 1_R)$$

chiral realization at $O(G_F p^2)$involves 2 LECs :

$$G_8, G_{27}$$

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \lambda L_\mu L^\mu \rangle + G_{27} F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3} L_{\mu 21} L^\mu_{13} \right) + \text{h.c.}$$

$$\lambda = (\lambda_6 - i\lambda_7)/2 \;, \qquad L_\mu = i U^\dagger D_\mu U$$

$$A_0 = F(M_K^2 - M_\pi^2)\sqrt{2}\,(G_8 + G_{27}/9)$$
$$A_2 = F(M_K^2 - M_\pi^2) 10\; G_{27}/9$$

implies

$$|G_8| \simeq 9.1 \cdot 10^{-6} \mathrm{GeV}^{-2}$$
$$|G_{27}| \simeq 0.5 \cdot 10^{-6} \mathrm{GeV}^{-2}$$
$$G_{27}/G_8 \simeq 1/18$$

theoretical estimates (in units $10^{-6}\mathrm{GeV}^{-2}$) :

	factorization $C_\pm = 1$	factorization $C_\pm(1\ \mathrm{GeV})$	exp.
G_8	1.1	2.1	9.1
G_{27}	1.1	0.8	0.5

other decays occurring at lowest order : $K \to 3\pi$

$\to$ only very qualitative agreement with exp. :
deviations $20 \div 30\%$ in amplitude $\simeq \frac{M_K^2}{(4\pi F_\pi)^2}$

CHPT : no other information at $O(G_F p^2)$
(Bremsstrahlung only)
$\rightarrow$ interesting physics starts at $O(G_F p^4)$

$K \rightarrow 2\pi, 3\pi$ at $O(G_F p^4)$

$\mathcal{L}_4^{\Delta S=1}$: 22 new LECs (octet)

predictive power ?

Actually 7 parameters (including G_8, G_{27}) for
12 observables $\rightarrow$ 5 relations (Kambor et al.)
for quadratic slope parameters in $K \rightarrow 3\pi$
(in units 10^{-8})

parameter	prediction	exp. value
ζ_1	-0.47 ± 0.18	-0.47 ± 0.15
ξ_1	-1.58 ± 0.19	-1.51 ± 0.30
ζ_3	-0.011 ± 0.006	-0.21 ± 0.08
ξ_3	0.092 ± 0.030	-0.12 ± 0.17
ξ_3'	-0.033 ± 0.077	-0.21 ± 0.51

Caveat : isospin violation not included

By-product : $|G_8|$ reduced by 30%
$\rightarrow$ right direction for $\Delta I = 1/2$ rule

Can chiral dynamics explain the $\Delta I = 1/2$ rule ?

Resummation of $\pi\pi$ rescattering via dispersion relations $\rightarrow$ right direction, but still sizable ambiguities
Truong; Pallante, Pich; Paschos; Buras et al.

Other approaches : $1/N_c$ expansion, lattice

Isospin breaking in $K \to 2\pi$

naive estimates

$O(m_u - m_d)$	$O(\alpha)$
$\dfrac{M_{K^0}^2 - M_{K^+}^2}{M_K^2} \sim 1.5\%$	$\dfrac{M_{\pi^+}^2 - M_{\pi^0}^2}{M_K^2} \sim 0.5\%$

Conclusions :

- 2 effects comparable in size
- isospin violation can be neglected in A_0
- possible strong enhancement in A_2 :

$$A_2^{\text{ind}}/A_2 \sim \frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2} \cdot \frac{A_0}{A_2} \sim 0.35$$

Application (E., Müller, Neufeld, Pich) :

π^0-η mixing and CP violation

relevant quantity (for $m_u \neq m_d$)

$$\Omega_{IB} := \frac{\mathrm{Im} A_2^{\mathrm{IB}}}{\omega \mathrm{Im} A_0} \qquad \omega := \mathrm{Re} A_2 / \mathrm{Re} A_0 = 1/22.1$$

contributes to CP-violating ratio ε'/ε as

$$\frac{\varepsilon'}{\varepsilon} \approx 13\, \mathrm{Im} V_{ts}^* V_{td} \left[B_6^{(1/2)} (1 - \Omega_{IB}) - 0.4 B_8^{(3/2)} \right]$$

Munich group (Buras et al.)

$B_6^{(1/2)}, B_8^{(3/2)}$: bag factors , $O(1)$

$O(p^2) = O[(m_u - m_d) p^0]$

$$\Omega_{IB} = \frac{2\sqrt{2}\varepsilon_{\pi^0\eta}^{(2)}}{3\sqrt{3}\omega} = 0.13$$

due to π^0-η mixing

$$\varepsilon_{\pi^0\eta}^{(2)} = \frac{\sqrt{3}(m_d - m_u)}{4(m_s - \hat{m})}$$

$O(p^4) = O[(m_u - m_d)p^2]$

η' contribution large

$$\Omega_{\eta+\eta'} = 0.25 \pm 0.08$$

Donoghue et al.; Buras, Gérard; Cheng; Lusignoli

Legitimate question :

Other contributions at this order ?

calculate matrix propagator in basis of
octet fields π_3, η_8

diagonalization gives rise to correction

$$\varepsilon_{\pi^0\eta} = \varepsilon^{(2)}_{\pi^0\eta} + \varepsilon^{(4)}_{\pi^0\eta}$$

with

$$\varepsilon^{(4)}_{\pi^0\eta}/\varepsilon^{(2)}_{\pi^0\eta} = \frac{128(M_K^2 - M_\pi^2)^2}{3F_\pi^2(M_\pi^2 - M_\eta^2)}[3L_7 + L_8^r(M_\rho)]$$
$$+ \quad \text{(small) loop corrections}$$

numerical analysis

L_7 due to η' exchange :

$$\varepsilon^{(4)}_{\pi^0\eta}(L_7)/\varepsilon^{(2)}_{\pi^0\eta} = 1.10$$

consistent with $\Omega_{\eta+\eta'} = 0.25 \pm 0.08$

HOWEVER

$$\varepsilon^{(4)}_{\pi^0\eta}(L^r_8(M_\rho))/\varepsilon^{(2)}_{\pi^0\eta} = -0.83$$

$\Rightarrow$ large cancellation !

Remark : L_8 dominated by $a_0(980)$ exchange

Total contribution of $O(p^4)$

in terms of

$$3L_7 + L^r_8(M_\rho) = (-0.25 \pm 0.25) \cdot 10^{-3}$$

$$\rightarrow \quad \Omega^{\pi^0\eta}_{IB} = 0.16 \pm 0.03$$

with $B^{(1/2)}_6, B^{(3/2)}_8$ from Munich group :

$\frac{\varepsilon'}{\varepsilon}$ increased by 21 %

additional isospin-violating contributions to Ω_{IB}
depend on weak LECs
model-dependent estimates $\rightarrow$

$$\Omega_{IB} \sim \Omega^{(2)}_{IB}$$

Wolfe, Maltman; Gardner, Valencia

Electromagnetic corrections in $K \to 2\pi$

to first order in $m_u - m_d$ and with $G_{27} = 0$:

$$\Rightarrow \quad A_{5/2} = 0$$

Roy equations $\to$

$$(\delta_0^0 - \delta_0^2)_{\rm Roy} = 45^\circ \;{}^{+5^\circ}_{-2^\circ}$$

from $K \to \pi\pi$ branching ratios :

either

$$A_{5/2} = 0 \qquad \Rightarrow \qquad (\delta_0^0 - \delta_0^2)_{K\to\pi\pi} = 57^\circ \pm 4^\circ$$

or

$$(\delta_0^0 - \delta_0^2)_{\rm Roy} \qquad \Rightarrow \qquad {\rm Re}A_{5/2} = -(0.17 \pm 0.07){\rm Re}A_{3/2}$$

$$({\rm Re}A_{5/2} \sim -\alpha{\rm Re}A_{1/2})$$

$\to$ 2.5 σ evidence for elm. corrections

Systematic CHPT analysis to

$$O(G_8 e^2 p^0) \quad \text{and} \quad O(G_8(m_u - m_d)p^2)$$

$\to$ 12 new LECs of $O(G_8 e^2 p^0)$(for $K \to \pi\pi$)

Cirigliano, Donoghue, Golowich : $O(e^2)$
Wolfe, Maltman : $O(m_u - m_d)$
E., Isidori, Müller, Neufeld, Pich : both

first attempt (Cirigliano et al.)

$$\mathrm{Re}A_{5/2} = (0.05 \pm 0.06)\mathrm{Re}A_{3/2}$$

disagrees with exp. determination

$$\mathrm{Re}A_{5/2} = -(0.17 \pm 0.07)\mathrm{Re}A_{3/2}$$

$\rightarrow$ more work needed

Rare Kaon Decays

rich experimental program
CERN, BNL, FNAL, KEK, DAΦNE

CHPT : except for Bremsstrahlung,
all amplitudes start at $O(G_F p^4)$

Classification :

i. Short-distance dominated

all relevant physics contained in LECs
→ little information from CHPT

Examples :

$$K \to \pi\nu\overline{\nu} \; , K_L \to \pi^+\pi^-\nu\overline{\nu}$$

ii. Decays without LECs of $O(G_F p^4)$

some vanish at $O(G_F p^4)$:

$$K_L \to \gamma\gamma \; , \; K_L \to \pi^0\pi^0\gamma$$

the others are completely calculable to
$O(G_F p^4)$(in terms of G_8 or G_{27})

Best measured examples :

$$K_L \to \pi^0\gamma\gamma \ , \ K_S \to \gamma\gamma$$

$K_L \to \pi^0\gamma\gamma$

$O(G_F p^4)$: pure one-loop amplitude
E., Pich, de Rafael (1987)

$\to$ characteristic spectrum in $M_{\gamma\gamma}$
confirmed in 1990 by NA31 (CERN) T

However :
theoretical rate too low

$$BR(K_L \to \pi^0\gamma\gamma) = \begin{cases} 0.7 \cdot 10^{-6} & O(G_F p^4) \\ (1.68 \pm 0.10) \cdot 10^{-6} & \text{PDG 2000} \end{cases}$$

Dominant contributions of $O(G_F p^6)$:
Cappiello, D'Ambrosio, Miragliuolo; Cohen, E., Pich; Kambor, Holstein

- unitarity corrections (disp. calculation)
- vector meson exchange (one relevant additional parameter a_V)

$\to$ correlation between rate and spectrum higher-order contributions tend to

decrease peak at large $M_{\gamma\gamma}$ T

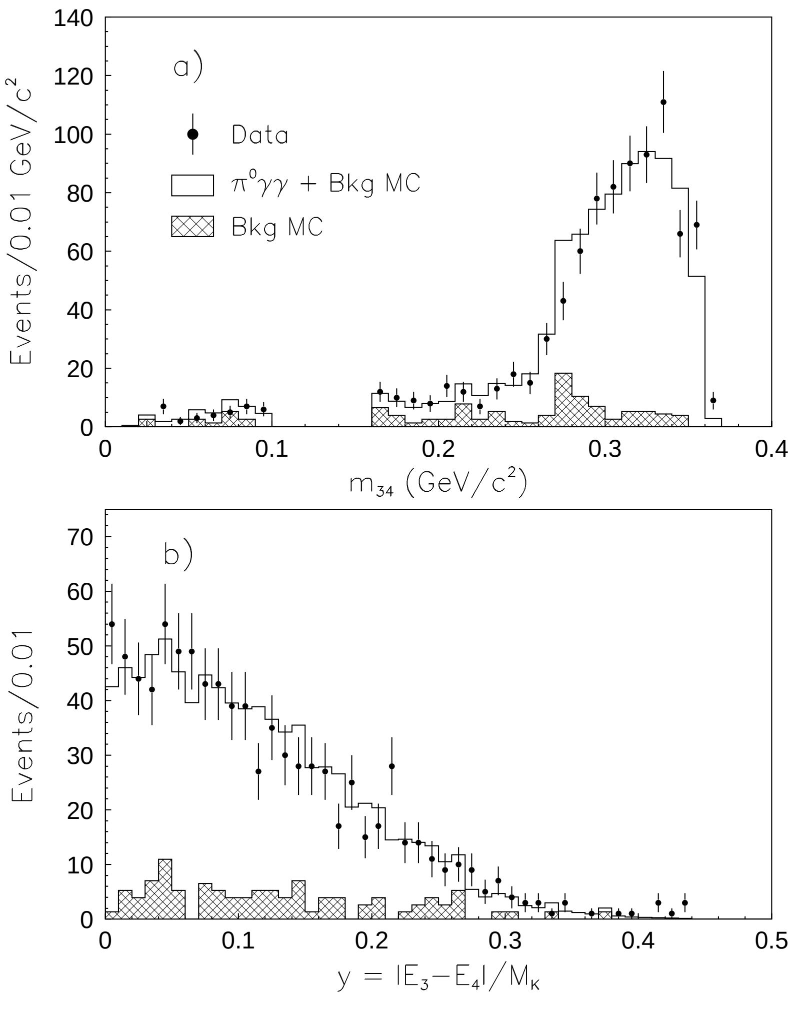
Recent analysis of KTeV (FNAL) :

fit $M_{\gamma\gamma}$ and $E_{\gamma 1} - E_{\gamma 2}$ distributions

$$\rightarrow \quad a_V = -0.72 \pm 0.08$$

reproduces experimental rate

$$BR(K_L \rightarrow \pi^0 \gamma\gamma) = \begin{cases} (1.53 \pm 0.10) \cdot 10^{-6} & O(G_F p^6) \\ (1.68 \pm 0.10) \cdot 10^{-6} & \text{PDG 2000} \end{cases}$$



Additional message :
with same kind of analysis, essentially

no corrections of $O(G_F p^6)$for $K_S \to \gamma\gamma$

$\to$ CHPT prediction of $O(G_F p^4)$ (D'Ambrosio, Espriu; Goity)practically unchanged

$$BR(K_S \to \gamma\gamma) = \begin{cases} 2.0 \cdot 10^{-6} & O(G_F p^4) \\ (2.4 \pm 0.9) \cdot 10^{-6} & \text{NA31} \end{cases}$$

iii. Decays with LECs of $O(G_F p^4)$

majority of decay channels
depend on

5	electric LECs	$N_{14}, \ldots, N_{18}$
4	magnetic LECs	$N_{28}, \ldots, N_{31}$

from the following table $\to$

- all electric, but only 3 combinations of magnetic LECs experimentally accessible
- many relations remain to be tested

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$
$\pi^0\gamma^*$ (S)	$\pi^0\pi^0\gamma^*$ (L)		$2N_{14}^r + N_{15}^r$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma$ (S)		"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma$ (S)	$\pi^+\pi^0\pi^0\gamma$	"
		$\pi^+\pi^-\pi^0\gamma$ (L)	"
		$\pi^+\pi^-\pi^0\gamma$ (S)	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$
	$\pi^+\pi^-\gamma^*$ (L)		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma^*$ (S)		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma$ (L)	$\pi^+\pi^-\pi^0\gamma$ (S)	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma$ (S)	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma$ (L)	$6N_{28} + 3N_{29} - 5N_{30}$

Unlike for strong LECs :
no established framework for predicting
the weak LECs N_i

$K \to \pi l^+ l^-$

2 different classes :

A. $K^\pm \to \pi^\pm l^+ l^-$, $K_S \to \pi^0 l^+ l^-$
dominated by single-photon exchange

B. $K_L \to \pi^0 l^+ l^-$
single-photon exchange CP-violating

General structure of $K \to \pi\gamma^* \to \pi l^+ l^-$:

$$A(K^i(k) \to \pi^i(p) l^+ l^-) = -\frac{e^2}{M_K^2 (4\pi)^2} W_i(z)(k+p)^\mu \bar{u}_l \gamma_\mu v_l$$

form factors $W_i(z)$ $(z = (k-p)^2/M_K^2;\ i = +, 0)$:

- $W_i(0)$ non-singular (in spite of photon pole)
- W_+, W_0 not related by chiral symmetry $\to$ $K_S \to \pi^0 l^+ l^-$ not predictable from $K^+ \to \pi^+ l^+ l^-$

general decomposition (D'Ambrosio et al.)

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

$$W_i^{\rm pol}(z) = a_i + b_i z + \ldots$$

$a_i(b_i)$ enter at $O(G_F p^{4(6)})$

normalization : $a_i = O(1)$ expected

order	$W_i^{\pi\pi}(z)$
$G_8 p^4$	simple pion loop
	E., Pich, de Rafael (1987)
$G_8 p^6$	dispersion relation using $A(K \to 3\pi)_{\rm exp}$
	D'Ambrosio, E., Isidori, Portolés (1998)

Motivation for dispersive calculation

BNL-E777 : approximate agreement with $O(G_8 p^4)$prediction

BNL-E787 : strong disagreement for μ/e ratio

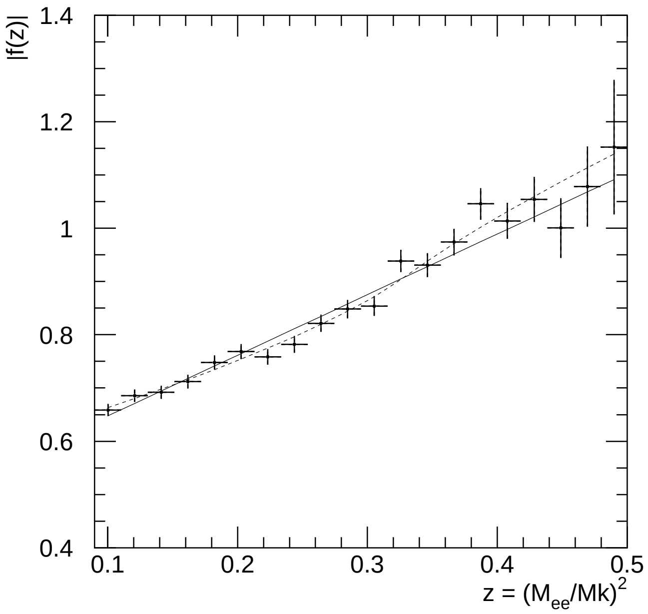
$R := \Gamma(K^+ \to \pi^+\mu^+\mu^-)/\Gamma(K^+ \to \pi^+e^+e^-)$

$$R = \begin{cases} 0.23 & O(G_8 p^4) \\ 0.167 \pm 0.036 & \text{BNL-E787} \end{cases}$$

Recent developments

high-statistics result for $K^+ \to \pi^+e^+e^-$

BNL-E865 : $\sim 10^4$ events



fit	$\chi^2/N_{\rm dof}$
linear in z	22.9/18
including $W_i^{\pi\pi}$	13.3/18

Conclusions :

- clear evidence for $W_+^{\pi\pi}$ as required by CHPT
- $a_+ = -0.59 \pm 0.01$, $b_+ = -0.65 \pm 0.04$
  $\rightarrow$ $O(G_F p^6)$effects big
- prediction for $l = \mu$:
  $BR(K^+ \rightarrow \pi^+\mu^+\mu^-) \simeq 8.7 \cdot 10^{-8}$

preliminary result from BNL-E865 :

$$BR(K^+ \rightarrow \pi^+\mu^+\mu^-) = (9.23 \pm 0.6 \pm 0.58) \cdot 10^{-8}$$

$K_S \rightarrow \pi^0 l^+ l^-$

$O(G_8 p^6)$prediction

$$BR(K_S \rightarrow \pi^0 e^+ e^-) \simeq 5|a_S|^2 \cdot 10^{-9}$$

Reason : pion loop negligible

($K_S \rightarrow \pi^+\pi^-\pi^0$ suppressed)

hardly sensitive to b_S

naive estimate $a_S \sim O(1) \quad \rightarrow$

within reach of NA48 and KLOE (DAΦNE)

moreover :

μ/e ratio reliably predicted $\quad \rightarrow \quad R \simeq 0.23$

$$K_L \rightarrow \pi^0 l^+ l^-$$

Recall : single-photon exchange CP violating

3 sources :

1. CP-conserving amplitude due to
   2γ exchange $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+ e^-$
   with new results for $K_L \rightarrow \pi^0 \gamma\gamma \quad \rightarrow$
   $BR(K_L \rightarrow \pi^0 e^+ e^-)_{\rm CPC} \lesssim 2 \cdot 10^{-12}$

2. Direct CP violation due to
   Z-penguin and W-box diagrams $\quad \rightarrow$
   $BR(K_L \rightarrow \pi^0 e^+ e^-)_{\rm CPV-dir} \sim 5 \cdot 10^{-12}$
   Buras et al.

3. Indirect CP violation due to mixing $\quad \rightarrow$
   $BR(K_L \rightarrow \pi^0 e^+ e^-)_{\rm CPV-ind} \sim$
   $3 \cdot 10^{-3} BR(K_S \rightarrow \pi^0 e^+ e^-)$

However :

CP-violating amplitudes interfere $\rightarrow$

$$B(K_L \rightarrow \pi^0 e^+ e^-)_{CPV} =$$

$$\left[15.3 a_S^2 - 6.8 \frac{\mathrm{Im}\lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\mathrm{Im}\lambda_t}{10^{-4}}\right)^2\right] \times 10^{-12}$$

with $\lambda_t = V_{td} V_{ts}^*$

Thus :

for $a_S \lesssim -0.5$ or $a_S \gtrsim 1.0 \quad \rightarrow$

$$B(K_L \rightarrow \pi^0 e^+ e^-)_{CPV} \gtrsim 10^{-11}$$

experimentally accessible in the near future

At the same time :

CP-conserving contribution negligible
(non-interfering !)

present exp. limit (KTeV)

$$B(K_L \rightarrow \pi^0 e^+ e^-) < 5.6 \cdot 10^{-10} \ (90\% \ \mathrm{CL})$$

Conclusions

- comprehensive treatment of all K decays
- $O(G_F p^6)$effects are being tested both in semileptonic and in nonleptonic decays
- systematic inclusion of isospin violation and electromagnetic corrections $\rightarrow$ essential for understanding $\Delta I = 1/2$ rule, CP violation
- better theoretical understanding of weak LECs (resonance effects) needed to interpret forthcoming results from precision expts.
- well-founded expectation

K decays will continue to provide information
on the SM at low energies
or even
on physics beyond