Applications of CHPT

not restricted to strong interactions of mesons

Inclusion of baryons

straightforward in principle

Recall : $\psi \xrightarrow{g\in G} h_\psi(g,\phi)\psi$

BUT

- baryons are not Goldstone particles
  - → there are no "soft" baryons
  - → interactions less constrained
- $\mathcal{L}_{\text{eff}} = \sum_n \mathcal{L}_n$ with

  n=1,2,3,...

  mesons only : n=2,4,6, ...
- baryon resonances often closer to threshold (Δ vs. ρ)
- nucleon mass m complicates chiral counting

$$\frac{\vec{p}}{4\pi F} \quad \sim \quad \frac{\vec{p}}{m}$$

suggests simultaneous expansion
($\vec{p}$: small three-momentum)

Traditional method : shift m from propagators to vertices

Heavy Baryon CHPT

Jenkins, Manohar (in analogy to HQET)

Idea : decompose nucleon field ($N_f = 2$) into "light" and "heavy" components and integrate out the latter

Exemplify for free nucleons :

$$\mathcal{L}_0 = \overline{\Psi}(i\partial\!\!\!/ - m)\Psi$$

introduce projectors

$$P_v^{\pm} = \frac{1}{2}(1 \pm v\!\!\!/) \ , \qquad v^2 = 1$$

in rest system with $v = (1, 0, 0, 0)$:
$P_v^{\pm}$ project on upper and lower components of Ψ

Def.: frame-dependent fields N_v , H_v

$$
\begin{aligned}
N_v(x) &= \exp[imv \cdot x] P_v^+ \Psi(x) \\
H_v(x) &= \exp[imv \cdot x] P_v^- \Psi(x)
\end{aligned}
$$

$$
\begin{aligned}
\rightarrow \quad \mathcal{L}_0 &= \overline{N_v} iv \cdot \partial N_v - \overline{H_v}(iv \cdot \partial + 2m) H_v \\
&+ \quad \text{mixed terms}
\end{aligned}
$$

integrating out "heavy " field H_v (mass=$2m$!) in the fully relativistic πN Lagrangian

$\rightarrow$ effective Lagrangian for N_v (and pions) with massless propagator

$$
\frac{iP_v^+}{v \cdot k + i\varepsilon} = O(p^{-1})
$$

$\rightarrow$ for one-nucleon processes, the chiral dimension is given by (exercise)

$$
\begin{aligned}
D_L &= 2L + 1 + \sum_n [(n-2)N_{n,0} + (n-1)N_{n,2}] \\
&\geq 2L + 1
\end{aligned}
$$

$N_{n,m}$: number of vertices from nth-order Lagrangian with m nucleon fields ($m = 0, 2$)

Remarks :

- Lorentz invariant amplitudes can be recovered uniquely (to the chiral order considered) from frame-dependent amplitudes
- expansion does not converge in some kinematic regions (discussion)

Alternative method :

Relativistic Baryon CHPT

Becher, Leutwyler (related work : Ellis, Tang)

Properties :

- manifest Lorentz invariance at every stage
- (infrared) regularization more involved than standard dim. regularization
- usually fewer diagrams, but calculations more cumbersome (full Dirac algebra)

Other heavy fields can be incorporated

meson resonances : $\rho, \ldots$

baryon resonances : $\Delta, \ldots$

Electroweak interactions

Green functions of quark currents not sufficient

Reason : strong ints. cannot be disentangled

Nonleptonic weak interactions

Procedure : OPE

first integrate out W and heavy quarks $\rightarrow$

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i + \text{h.c.}$$

$C_i(\mu)$: Wilson coefficients

Q_i : four-quark operators (dim=6)

for general CHPT analysis :

transformation properties under G

$$\mathcal{H}_{\rm eff}^{\Delta S=1} \sim (8_L, 1_R) + (27_L, 1_R)$$

WANTED

Effective Lagrangian for an effective Hamiltonian

Method : spurion fields

e.g., for octet Hamiltonian :

spurion $\lambda(x)$ transforming as octet field

$$\lambda(x) \stackrel{g}{\to} g_L \lambda(x) g_L^{-1}$$

– construct general chiral Lagrangian
linear in $\lambda(x)$

– project out octet part by setting

$$\lambda(x) = \frac{1}{2}(\lambda_6 - i\lambda_7)$$

More involved for

Electromagnetic interactions

major difference : nonlocal at low energies

Necessary ingredients :

- replace external vector field by

$$v_\mu \to v_\mu - eQA_\mu$$

  with quark charge matrix Q, photon field A

- photon kinetic term (+ gauge fixing)

- restricted to $O(e^2)$:
  add chiral Lagrangian
  bilinear in spurion fields $Q_L(x), Q_R(x)$

with transformation properties

$$Q_A(x) \xrightarrow{g} g_A Q_A(x) g_A^{-1}, \qquad A = L, R$$

setting

$$Q_A(x) = Q = \text{diag}(2/3, -1/3, -1/3)$$

$\rightarrow$ **effective chiral Lagrangian of $O(e^2)$**

Altogether : $\mathcal{L}_{\text{eff}}^{\text{SM}}$ T

List of applications

Mesons

- strong interactions (+ ext. γ, W)

  meson-meson scattering, $\gamma\gamma \rightarrow \pi\pi$
  meson masses ($\rightarrow m_q$), meson form factors
  semileptonic (radiative) decays
  η decays, $P \rightarrow l^+ l^-$

- nonleptonic weak interactions

  $K \rightarrow 2\pi, 3\pi$, rare K decays

Effective chiral Lagrangian $\mathcal{L}_{\rm eff}^{\rm SM}$ of the SM

(Meson and single-baryon sectors)

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}^{\rm odd}(0) + \mathcal{L}_{G_F p^2}^{\Delta S=1}(2)$ $+ \mathcal{L}_{e^2 p^0}^{\rm em}(1) + \mathcal{L}_{G_8 e^2 p^0}^{\rm emweak}(1)$ $+ \mathcal{L}_{p}^{\pi N}(1) + \mathcal{L}_{p^2}^{\pi N}(7)$ $+ \mathcal{L}_{G_8 p^0}^{MB,\Delta S=1}(2) + \mathcal{L}_{G_8 p}^{MB,\Delta S=1}(8)$ $+ \mathcal{L}_{e^2 p^0}^{\pi N,{\rm em}}(3)$	$L = 0$
$+ \underline{\mathcal{L}_{p^4}^{\rm even}(10)} + \underline{\mathcal{L}_{p^6}^{\rm odd}(32)} + \underline{\mathcal{L}_{G_8 p^4}^{\Delta S=1}(22)}$ $+ \underline{\mathcal{L}_{e^2 p^2}^{\rm em}(14)} + \underline{\mathcal{L}_{G_8 e^2 p^2}^{\rm emweak}(15)} + \underline{\mathcal{L}_{e^2 p}^{\rm leptons}(5)}$ $+ \underline{\mathcal{L}_{p^3}^{\pi N}(23)} + \underline{\mathcal{L}_{p^4}^{\pi N}(114)} + \underline{\mathcal{L}_{G_8 p^2}^{MB,\Delta S=1}(?)}$ $+ \underline{\mathcal{L}_{e^2 p}^{\pi N,{\rm em}}(8)}$	$L = 1$
$+ \underline{\mathcal{L}_{p^6}^{\rm even}(90)}$	$L = 2$

$N_f = 3$ (except for πN)

even (odd) intrinsic parity :

mesonic Lagrangian without (with) ε tensor

underlined : completely renormalized Lagrangians

- virtual photons

elm. mass differences
elm. corrections (isospin violation) for strong and weak processes (CP violation)

- leptons and photons

elm. corrections for semileptonic decays

Baryons and mesons

- strong interactions (+ ext. γ, W)

$\pi N \to \pi(\pi)N, KN \to KN$, σ terms
photo-, electro-, ν-production of π, K, η
baryon form factors, Compton scattering
magnetic moments, polarizabilities

- nonleptonic weak interactions

(radiative) weak baryon decays

- virtual photons

elm. corrections and isospin violation

- nuclear physics [not included in $\mathcal{L}_{\text{eff}}^{\text{SM}}$]

NN interactions, few-nucleon systems

Pion-Pion Scattering

Motivation

- the fundamental scattering process of CHPT for $N_f = 2$
- sensitive to mechanism of SCSB
- new experimental activity

BNL-E865	K_{e4} $(K \to \pi\pi e\nu_e)$
NA48 (CERN)	”
KLOE (DAΦNE)	”
DIRAC (CERN)	pionium

Scattering amplitude in CHPT

isospin limit : $m_u = m_d \;, \alpha = 0$

kinematics T

Kinematics

$$\pi^a(p_a) + \pi^b(p_b) \to \pi^c(p_c) + \pi^d(p_d)$$

Scattering amplitude ($m_u = m_d$)

$$\begin{aligned} T_{ab,cd}(s,t,u) &= \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(t,s,u) \\ &+ \delta_{ad}\delta_{bc}A(u,t,s) \end{aligned}$$

Bose symmetry $\quad \to \quad A(s,t,u) = A(s,u,t)$

$$\begin{aligned} s &= (p_a + p_b)^2 = 4(q^2 + M_\pi^2) \\ t &= (p_a - p_c)^2 = -2q^2(1 - \cos\theta) \\ u &= (p_a - p_d)^2 = -2q^2(1 + \cos\theta) \end{aligned}$$

$q\ ,\theta$: CM momentum, scattering angle

$T^I(s,t)$: amplitudes with definite isospin I in s-channel

$$\begin{aligned} T^0(s,t) &= 3A(s,t,u) + A(t,u,s) + A(u,s,t) \\ T^1(s,t) &= A(t,u,s) - A(u,s,t) \\ T^2(s,t) &= A(t,u,s) + A(u,s,t) \end{aligned}$$

Partial wave expansion

$$T^I(s,t) = 32\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) t_l^I(s)$$

$t_l^I(s)$: partial wave amplitudes

elastic region $(4M_\pi^2 \leq s \leq 16M_\pi^2)$:

$$t_l^I(s) = (1 - \frac{4M_\pi^2}{s})^{-1/2} \exp i\delta_l^I(s) \sin \delta_l^I(s)$$

$\delta_l^I(s)$: phase shifts

expansion near threshold :

$$\mathrm{Re}\, t_l^I(s) = q^{2l}\{a_l^I + q^2 b_l^I + O(q^4)\}$$

threshold parameters

a_l^I : scattering lengths

b_l^I : slope parameters ($\sim$ effective ranges)

$O(p^2)$ $(L = 0)$

Standard CHPT

$$A_2(s,t,u) = \frac{s - M_\pi^2}{F_\pi^2}$$ Weinberg

together with ($2\hat{m} := m_u + m_d$)

$$r := \frac{m_s}{\hat{m}} = r_2 := \frac{2M_K^2}{M_\pi^2} - 1 \simeq 26$$

Generalized CHPT

more parameters already at lowest order

$$6 \leq r \leq r_2 \simeq 26$$ Stern et al.

$$A_2^{\text{GCHPT}}(s,t,u) = \frac{s - \frac{4}{3}M_\pi^2}{F_\pi^2} + \alpha \frac{M_\pi^2}{3F_\pi^2}$$

$$\alpha = 1 + \frac{6(r_2 - r)}{r^2 - 1}\ , \qquad \alpha \geq 1$$

S-waves especially sensitive to α :

a_0^0	r	α	B
0.16	26	1	large
0.26	10	2	small

$O(p^4)$ $(L \leq 1)$ standard CHPT

Gasser, Leutwyler

$$F_\pi^4 A_4(s,t,u) = c_1 M_\pi^4 + c_2 M_\pi^2 s + c_3 s^2 + c_4 (t-u)^2 + F_1(s) + G_1(s,t) + G_1(s,u)$$

F_1, G_1 : 1-loop functions

$c_1, \ldots, c_4$: contain ren. LECs $l_i^r(\mu)$ $(i = 1, \ldots, 4)$ and chiral logs (first power)

many observables dominated by chiral logs :

	$O(p^2)$	$O(p^4)$
a_0^0	0.16	0.20

most of the difference due to chiral logs $(\mu \simeq M_\rho)$

$O(p^6)$ $(L \leq 2)$

Dispersive calculation

Knecht, Moussallam, Stern, Fuchs

$A(s,t,u)$ calculable up to subtraction polynomial

$$[b_1 M_\pi^4 + b_2 M_\pi^2 s + b_3 s^2 + b_4 (t-u)^2]/F_\pi^4 + [b_5 s^3 + b_6 s (t-u)^2]/F_\pi^6$$

high-energy data ($E \geq 0.8$ GeV) $\rightarrow$ b_3, b_4, b_5, b_6
left with 2 free parameters

Field theoretic calculation

Bijnens, Colangelo, E., Gasser, Sainio

diagrams with $L = 0, 1, 2$ T

$\rightarrow$ reproduce analytically nontrivial part

$\rightarrow$ $b_i(M_\pi/F_\pi, M_\pi/\mu; l_i^r(\mu), r_i^r(\mu))$

$r_i^r(\mu)$ $(i = 1, \ldots, 6)$: LECs of $O(p^6)$

Advantages

- full infrared structure (chiral logs)
- dependence on LECs $\rightarrow$ comparison with other processes
- dependence on m_q (via M_π^2) $\rightarrow$ lattice QCD

Results

$r_i^r(\mu)$ estimated from meson resonance exchange
$\rightarrow$ small uncertainties for low partial waves

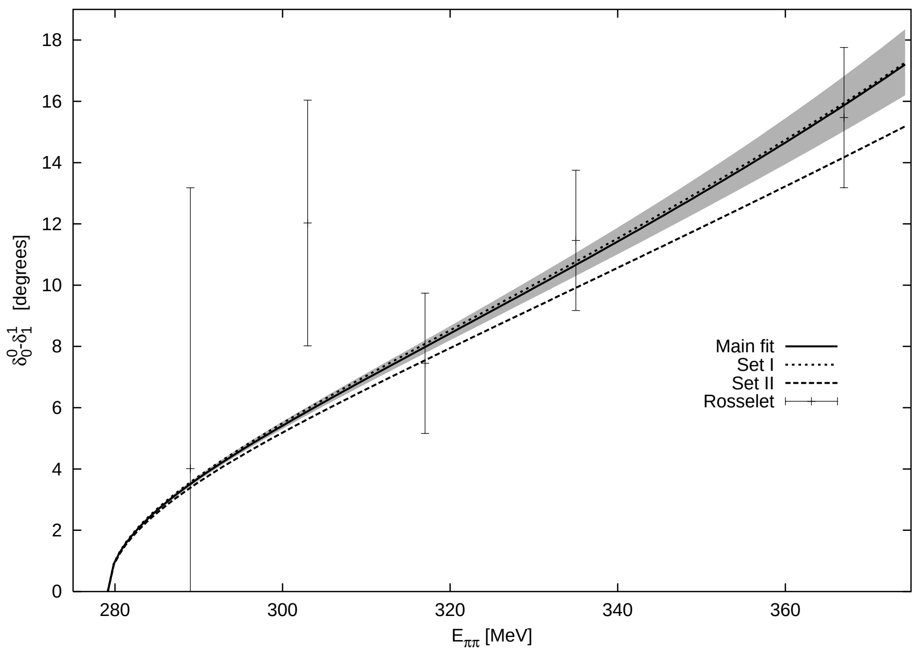
much more sensitive to $l_i^r(\mu)$ (LECs of $O(p^4)$)

set I : 1995 values

set ABT : new $O(p^6)$ fit (Amoros et al.)

phase shift difference $\delta_0^0 - \delta_1^1$

Amoros, Bijnens, Talavera



Most recent developments

A. Preliminary new results
from BNL-E865 (K_{e4}) T

B. New analysis of Roy equations
Ananthanarayan, Colangelo, Gasser, Leutwyler

Step 1

low partial waves (S, P) from dispersion relations (Roy equations) with input

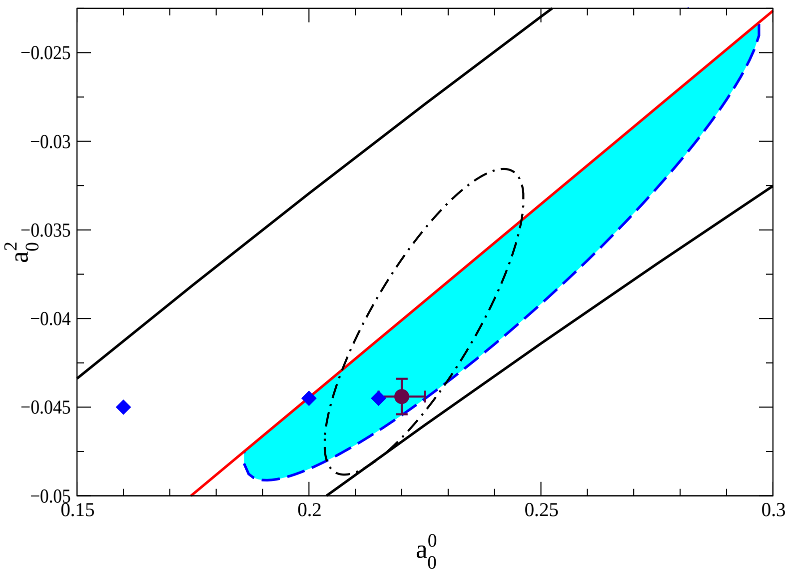
i. high-energy data ($E \geq 0.8$ GeV)

ii. scattering lengths a_0^0, a_0^2
(subtraction constants)

$\rightarrow$ amazingly precise predictions for phase shifts and remaining threshold parameters in terms of a_0^0, a_0^2

However :

even with new BNL-E865 results $\rightarrow$
allowed domain in ($a_0^0 - a_0^2$)-plane rather large



Step 2

Colangelo, Gasser, Leutwyler

match Roy and $O(p^6)$ CHPT solutions
at unphysical point $s = 0$

Reason : chiral expansion converges
much better at $s = 0$ than
at the physical threshold $s = 4M_\pi^2$

Input :

LECs of $O(p^4)$: l_3^r, l_4^r (main sensitivity)

LECs of $O(p^6)$: $r_1^r, r_2^r, r_3^r, r_4^r$

Threshold parameters for $\pi\pi$ scattering

	$O(p^6)$ set ABT	$O(p^6)$ set I	Roy
a_0^0	0.219 ± 0.005	0.222	0.220 ± 0.005
$-10a_0^2$	0.420 ± 0.010	0.420	0.444 ± 0.010
b_0^0	0.279 ± 0.011	0.282	0.276 ± 0.006
$-10b_0^2$	0.756 ± 0.021	0.729	0.803 ± 0.012
$10a_1^1$	0.378 ± 0.021	0.404	0.379 ± 0.005
$10^2 b_1^1$	0.59 ± 0.12	0.83	0.567 ± 0.013
$10^2 a_2^0$	0.22 ± 0.04	0.28	0.175 ± 0.003
$10^3 a_2^2$	0.29 ± 0.10	0.24	0.170 ± 0.013

Output :

$$a_0^0, a_0^2, \quad l_1^r, l_2^r, \quad r_5^r, r_6^r$$

Conclusions

- chiral expansion "converges"
- many observables dominated by chiral logs (especially S-waves)
- good agreement with all available data via Roy equations
- very solid prediction of standard CHPT :

$$\begin{aligned} a_0^0 &= 0.220 \pm 0.005 \\ a_0^2 &= -0.0444 \pm 0.0010 \end{aligned}$$

- However :
  depends on $O(p^4)$ determination of l_3, l_4
  which GCHPT may and does question ;
  nevertheless :

  $$a_0^0 < 0.25 \text{ from Roy equations alone}$$

- 2nd caveat : at this level of accuracy, isospin violation and elm. corrections must be included; partly available for $\pi\pi \to \pi\pi$ but not (yet) for K_{e4}

Isospin Violation

2 sources in SM :

i. $m_u \neq m_d$

ii. electroweak interactions
for $G_F = 0$: electromagnetic corrections

theoretical and experimental level of accuracy
$\rightarrow$ must include isospin-violating effects

Example : $\pi\pi \to \pi\pi$ at $O(p^2)$

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} = \begin{cases} 0.159 & M_{\pi^+} \\ 0.149 & M_{\pi^0} \end{cases}$$

difference comparable to $O(p^6)$ corrections !

Leading $\Delta I \neq 0$ effects

$$\mathcal{L}_2 + \mathcal{L}_4 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + \ldots - \frac{l_7}{16}\langle \chi U^\dagger - \chi^\dagger U\rangle^2$$

l_7 : tiny contribution $\sim (m_u - m_d)^2$ to $M_{\pi^0}^2$

$\rightarrow$ $m_u \neq m_d$ does not influence
$\pi\pi \rightarrow \pi\pi$ to $O(p^4)$

Leading electromagnetic corrections of $O(e^2p^0)$

$$\mathcal{L}^{\text{em}}_{e^2p^0} = e^2 Z F^4 \langle QUQU^\dagger \rangle$$

$$\rightarrow \quad \Delta M_\pi^2 = M_{\pi^+}^2 - M_{\pi^0}^2 = 2e^2 Z F^2 \quad , \qquad Z \simeq 0.8$$

no contribution to scattering amplitude
(except via M_π^2)

genuine leading electromagnetic corrections are

$$O(e^2p^2)$$

Pionic atoms

DIRAC (CERN) : measure lifetime of elm. bound π^+-π^- "atom" in ground state to 10% accuracy

$$\Gamma = \Gamma_{2\pi^0} + \Gamma_{2\gamma} + \ldots \qquad \frac{\Gamma_{2\gamma}}{\Gamma_{2\pi^0}} \simeq 4 \times 10^{-3}$$

Exercise : calculate binding energy

$$E_B = -M_{\pi^+}\alpha^2/4 \quad \text{and}$$

three-momentum of π^0 in final state

$$p^* = (M_{\pi^+}^2 - M_{\pi^0}^2 - M_{\pi^+}^2\alpha^2/4)^{1/2}$$

With amplitude in isospin limit :

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^* (a_0^0 - a_0^2)^2 \;\rightarrow\; \tau \simeq 3 \times 10^{-15} s$$

Deser, Goldberger, Baumann, Thirring

$a_0^0 - a_0^2$: scattering length for $\pi^+\pi^- \rightarrow \pi^0\pi^0$

$\rightarrow$ DIRAC aims to measure $|a_0^0 - a_0^2|$ to 5%

$\rightarrow$ crucial test for standard CHPT

Isospin violating corrections

Exercise : $A(\pi\pi \rightarrow \pi\pi)$ does not contain terms linear in $m_u - m_d$ in QCD

$\rightarrow$ leading corrections are $O(\delta)$ with $\delta = \alpha$ or $(m_u - m_d)^2$

Gall, Gasser, Lyubovitskij, Rusetsky

EFT technique : superior to

nonrelativistic potential

Bethe-Salpeter

...

Method

- $N_f = 2 +$ photons
- CHPT $\rightarrow$ nonrelat. effective Lagrangian (à la Caswell, Lepage) : elm. interaction only via Coulomb potential (to order required)
- calculate $E_B, \Gamma_{2\pi^0}$ and match to relativistic scattering amplitude with $O(\delta)$ corrections (Knecht, Urech)

corrected rate :

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^* A^2 (1+K)$$

$$A = -\frac{3}{32\pi}\mathrm{Re}A_{\mathrm{thr}}(\pi^+\pi^- \rightarrow \pi^0\pi^0) + o(\delta)$$

A_{thr} : relativistic on-shell amplitude $\rightarrow$

$$A = a_0^0 - a_0^2 + h_1(m_u - m_d)^2 + h_2\alpha + o(\delta)$$

h_1 : $O(\hat{m})$ $\rightarrow$ negligible

h_2 : $Z\times$ LECs of $O(p^4)$

LECs of $O(e^2p^2)$

h_i : isospin limit with M_{π^+}

numerically :

$$A = a_0^0 - a_0^2 + \varepsilon \ , \qquad \varepsilon = (0.58 \pm 0.16) \times 10^{-2}$$

additional corrections :

$$\begin{aligned} K &= \frac{\Delta M_\pi^2}{9M_\pi^2}(a_0^0 + 2a_0^2)^2 \\ &- \frac{2\alpha}{3}(\ln \alpha - 1)(2a_0^0 + a_0^2) + o(\delta) \\ &= 1.07 \times 10^{-2} \end{aligned}$$

Final result

$$\frac{\Gamma_{2\pi^0} - \Gamma_{2\pi^0}^{\mathrm{LO}}}{\Gamma_{2\pi^0}^{\mathrm{LO}}} = \underbrace{\frac{2\varepsilon}{a_0^0 - a_0^2}}_{0.047} + \underbrace{K}_{0.011} = 0.058$$

Remarks :

- general result valid to all orders in CHPT (for $\alpha = 0, m_u = m_d$) $\rightarrow$ a_l^I to $O(p^6)$
- with $p^* = O(\delta^{1/2})$ $\rightarrow$

  $\Gamma_{2\pi^0}$ calculated to $O(\delta^{9/2})$

  whereas $\Gamma_{2\gamma} = \frac{\alpha^5}{4} M_{\pi^+} = O(\delta^5)$